Further analysis with metric-affine $f(R)$ gravity

Rafid Hasan Dejrah

rafid.dejrah@gmail.com | **In**: Rafid Hasan Dejrah | www.rafid-hasan.com
Physics Department
Ankara University, Turkey

August 19, 2024

Background and Motivation

Why this topic:

- If we check our main goal, and set back for a while!
- Basically we need some presentations of this category.
- It does really root in even unimaginable levels.
- For a batter understanding of our universe.

Introduction and that is why ···:

- Studying cosmology and pontificating through it has been our horizon, especially lately.
- As we gain more knowledge, many obstacles come across our trajectory.
- Some of these problems is: the late Universe acceleration.
- Many attmepts has been proposed. E.g., models that requires exotic sources AND; on the other hand, modified gravity via non-Einsteinian dynamics.
- The Hubble issue.

$f(R)$:

- GR had a wide range of acceptance then, but it took physicists only 4 year to start questioning its uniqueness!
- Early vs. Late Universe.
- Some modification on the Einstein-Hilbert action.
- It is not that easy task, the competition is hard.
- There are mainly two variational principles to derive Einstein's equations from the Einstein-Hilbert action; Metric variation & Palatini variation.

metric-affine $f(R)$:

- But there is a third one; when the assumption of the Independence of matter action from the connection → **metric-affine $f(R)$ gravity** which makes it the most general!

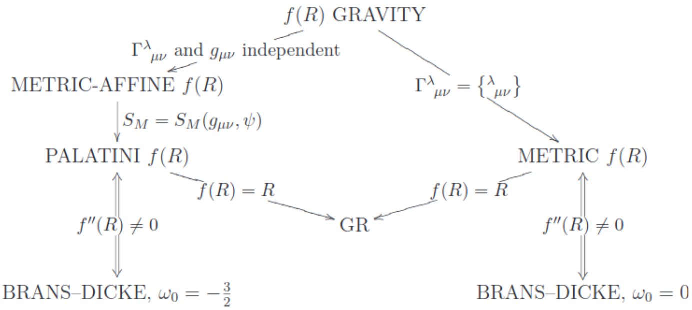
$$S_{MA} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}f(\mathscr{R}) + S_M(g_{\mu\nu}, \Gamma^{\lambda}_{\mu\nu}, \psi)$$

- where $\kappa \equiv 8\pi G$, G is the gravitational constant, g is the determinant of the metric,ψ collectively denotes the matter fields, $\mathscr{R} = g^{\mu\nu}\mathscr{R}_{\mu\nu}$.
- We need **hypermomentum** to mimics the definition of the stress-energy tensor as following

$$\Delta^{\mu\nu}_{\lambda} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta\Gamma^{\lambda}_{\mu\nu}}$$

- The way is long from here ···

Illustration:



[5]

SNOLAB · Laurentian University / Université Laurentienne · Arthur B. McDonald Canadian Astroparticle Physics Research Institute

The Horizons:

- It is the most general case of $f(R)$ for its enriched phenomenology, e.g., matter-induced, non-metricity, and torsion, which comes quite naturally since it is introduced by particles with spin.
- It is not a metric theory, hence the name! $\rightarrow T^{\mu\nu}$ is not divergence-free with respect to the covariant derivative defined with the Levi-Civita connection (nor with $\bar{\nabla}_{\mu}$)
- The physics meaning of the last statement is questionable and **further analysis** *is needed* since in the metric-affine gravity $T_{\mu\nu}$ does not really carry the usual stress-energy tensor, and we already have the hypermomentum which describes matter characteristics.
- Viability, representations, and Post-Newtonian limits

SNOLAB LaurentianUniversity UniversitéLaurentienne Arthur B. McDonald Canadian Astroparticle Physics Research Institute

[1] A. S. Eddington.
The Mathematical Theory of Relativity.
The University Press, Cambridge [Eng.], 1923.

[2] C. W. Misner, K. S. Thorne, and J. A. Wheeler.
Gravitation.
W. H. Freeman, San Francisco, 1973.

[3] G. Montani, M. DeăAngelis, F. Bombacigno, and N. Carlevaro.
Metric f(r) gravity with dynamical dark energy as a scenario for the hubble tension.
Monthly Notices of the Royal Astronomical Society: Letters, 527(1):L156L161, Oct. 2023.

[4] G. Sardanashvily.
International journal of geometric methods in modern physics: Preface.
6:v–viii, 12 2009.

[5] T. P. Sotiriou and V. Faraoni.
Reviews of Modern Physics, 82(1):451497, Mar. 2010.

[6] T. P. Sotiriou and S. Liberati.
The metric-affine formalism off(r) gravity.
Journal of Physics: Conference Series, 68:012022, May 2007.

[7] R. M. Wald.
General Relativity.
Chicago Univ. Pr., Chicago, USA, 1984.

[8] S. Weinberg.
Cosmology.
2008.

[9] H. Weyl.
A New Extension of Relativity Theory.
Annalen Phys., 59:101–133, 1919.

Thanks a bundle !

Don't hesitate reaching me out for any questions or collaboration

Rafid Hasan Dejrah

rafid.dejrah@gmail.com | **In**: Rafid Hasan Dejrah | `www.rafid-hasan.com`

Further analysis with metric-affine $f(R)$ gravity

Rafid Hasan Dejrah

rafid.dejrah@gmail.com | **In**: Rafid Hasan Dejrah | www.rafid-hasan.com
Physics Department
Ankara University, Turkey

August 19, 2024