

The Standard Model – Part 2: "This Way to Gold"

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Queen's
UNIVERSITY



PROGRAMME

- Part 1:
"Cut the Shackles ...
Changed My Name"
- Part 2:
"This Way to Gold"
- Part 3:
"I Came So Far to Get Lost at
Sea..."

*"So I cut the shackles and changed my name
And I shed my past like skin on a snake
But I came so far to get lost at sea
Oh, where the hell am I supposed to be?"*

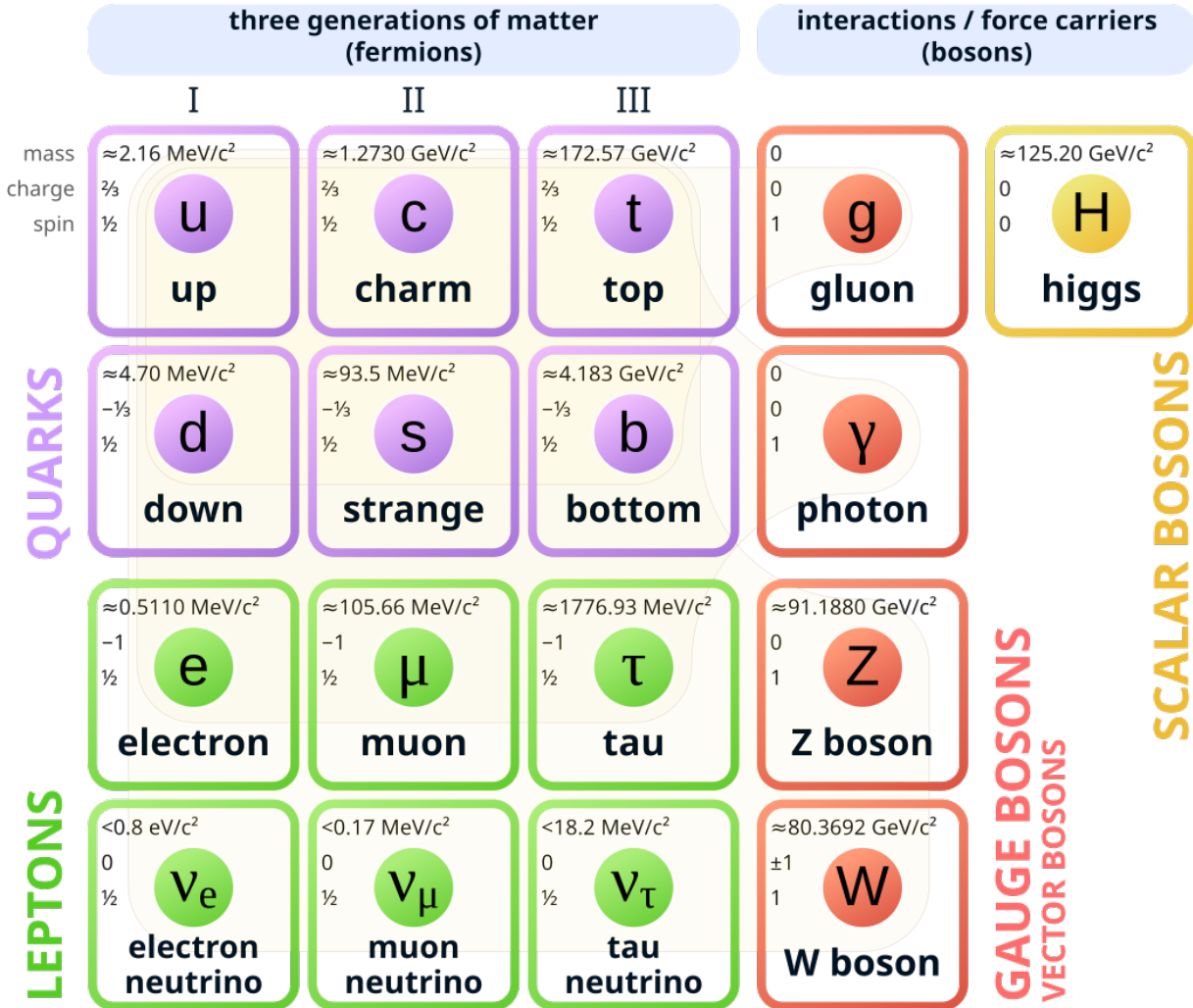
*And the sirens scream down every road
While the signs light up, 'This way to gold'
But I'm attached to my worst enemy
Oh, who the hell am I supposed to be?"*

Alice Merton, "Run Away Girl"



PART 2
This Way to Gold

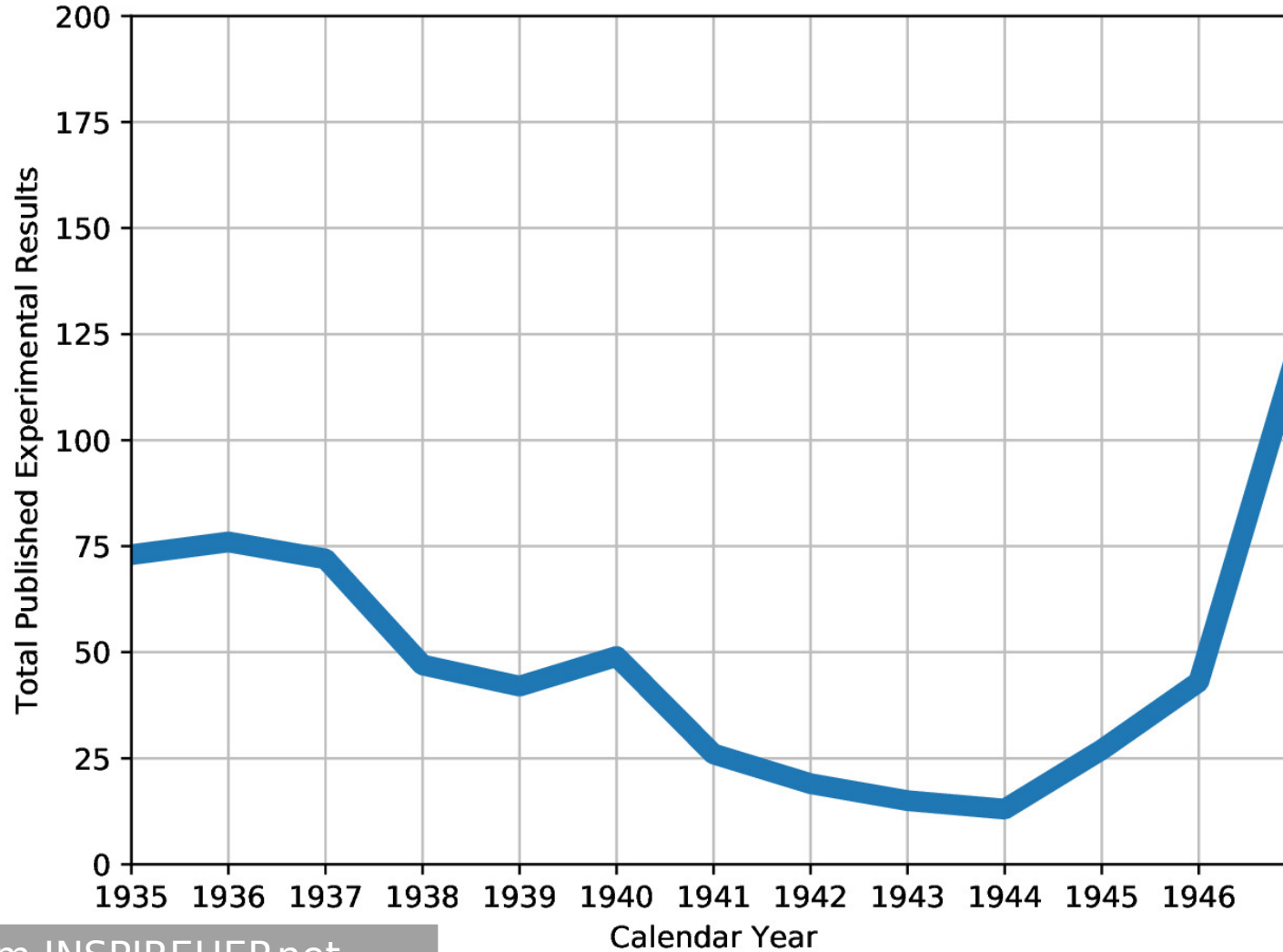
Standard Model of Elementary Particles



The modern understanding, based on a century of discovery.

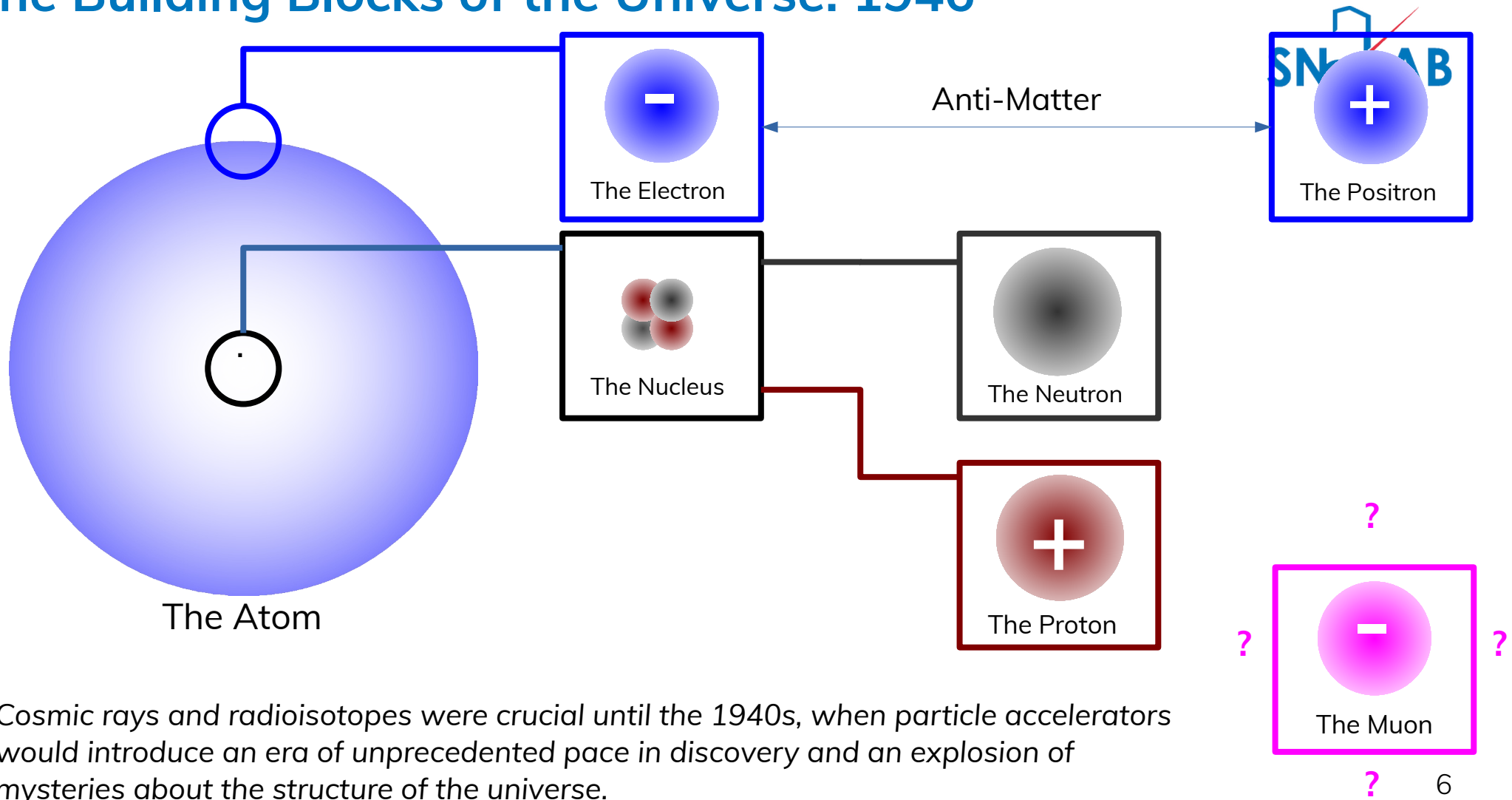
- Two families of matter particles, describable by quantum mechanics.
- Three distinct interactions (forces), also describable by quantum mechanics.
- Matter “charged” under forces → determines how they interact.
- Mathematical symmetries whose obedience/breaking defines the character of physical behaviour.

The Effect of WWII on Experimental Physics Publications



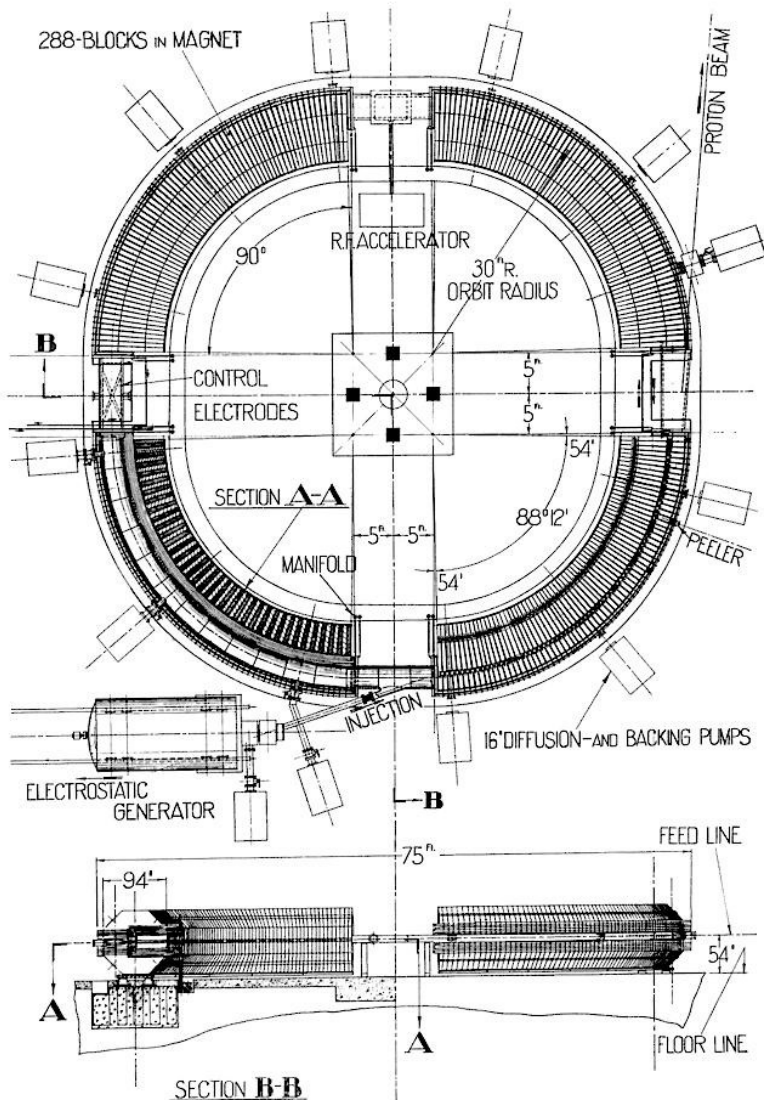
Data from INSPIREHEP.net

The Building Blocks of the Universe: 1946



Cosmic rays and radioisotopes were crucial until the 1940s, when particle accelerators would introduce an era of unprecedented pace in discovery and an explosion of mysteries about the structure of the universe.

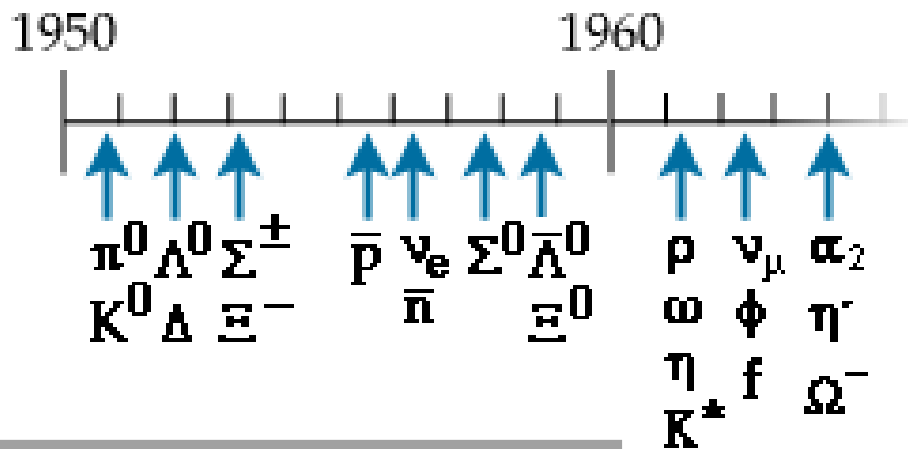
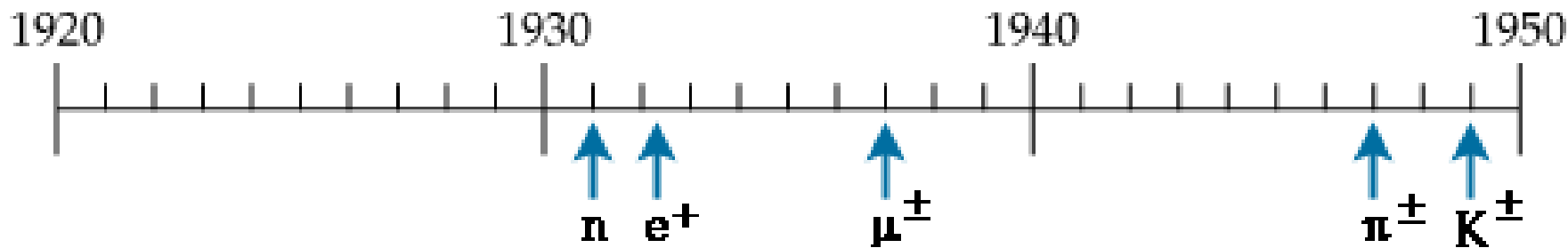
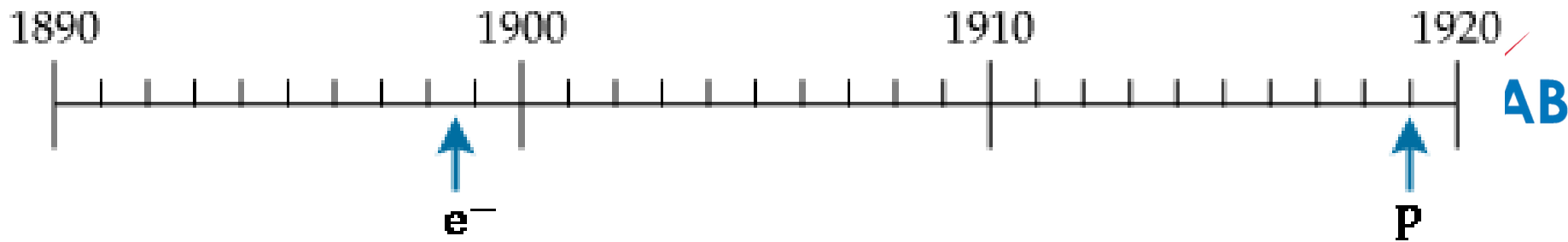
The Beginning of the Accelerator Era



- “Particle Accelerators” were not new – for example, Ernest Lawrence at Berkeley made his name building a novel compact circular particle accelerator before participating in the Manhattan Project.
- Accelerators are like hammers, microscopes, and time machines all rolled into one: you can smash matter to pieces and study the pieces; higher energies let you probe shorter distances in space; and high energies recreate conditions like those near the beginning of the universe.

$$E = hf = \hbar \omega, \quad p = h/\lambda = \hbar k$$

- Brookhaven Laboratory in NY began operating the world’s most powerful accelerator in 1950, the “Cosmotron” - conceived to be capable of providing protons with 1 billion eV of energy.



The beginning of the Age of Accelerators brought an explosion of discoveries and confusion. We now call the particles discovered between 1947 and 1974 the “hadrons”.

The new particles seemed to have nothing to do with the Atom. What is the connection to the Atom and why is the universe this way?

*You solve a problem by
predicting and looking
for a single new particle.*

You find 15 new particles.

You have two problems.



The Messiness of Sorting Out Nature: A Peek at the Hadrons

Making Sense of Nucleons

From the perspective of electromagnetism, the proton and neutron could not be more different. But EM doesn't result in the existence of protons and neutrons – some other force, assumed to be strong and limited to the nuclear realm (a size scale around $1\text{fm} = 10^{-15}\text{m}$), results in the nucleus. It was intriguing to physicists in the 1930s that the proton and neutron masses were so similar:

$$m_p = 938.27208943 \text{ MeV}/c^2$$

$$m_n = 939.56542052 \text{ MeV}/c^2$$



Heisenberg introduces the concept of what Wigner will later dub “Isospin” to try to explain this similarity. If protons and neutrons are distinct “isospin” states of some more fundamental underlying particle (a nucleon), then one could assign values to the proton ($I=+1/2$) and neutron ($I=-1/2$), such that $Q = e(I + 1/2)$ yields the charge of the nucleon.

While this was the wrong fundamental idea, this approach gets applied to a flurry of new particles discovered in the 1940s-1960s and ultimately leads to the currently accepted understanding encoded in the Standard Model.

Quantum Numbers

We are familiar with quantum numbers labeling atomic states, which are linked to the spin and orbital angular momentum states of electrons in the atom. However, additional observations in the early-mid 1900s indicated there were additional quantum numbers in nature.

Consider possible decays of the muon:

Possible Decay	Observed?
$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$	✓
$\mu^- \rightarrow e^- \gamma$	✗
$\mu^- \rightarrow e^- e^+ e^-$	✗



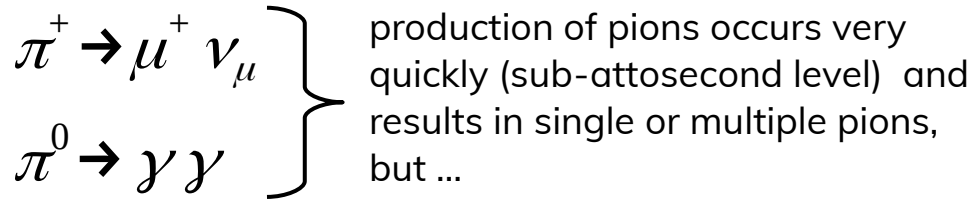
Implications: muons and electrons might have some new conserved quantum number associated with them. **Lepton number (L)** was born from this, with a distinct number for electron-type, muon-type, and tau-type particles.

Baryon Number

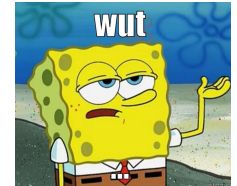
A similar set of observations with protons, neutrons, and related particles led to the proposal of **Baryon Number (B)**.

Pions^(*) and Kaons

The newly discovered mesons (intermediate in mass between electrons and protons ... at least for a time) were unstable and short-lived. Their mass scale is $\sim 100 \text{ MeV}/c^2$ and the dominant decay modes:



Particle	Mass (MeV/c ²)	Lifetime (s)
π^\pm	139.57039	2.6×10^{-8}
π^0	134.9768	8.5×10^{-17}



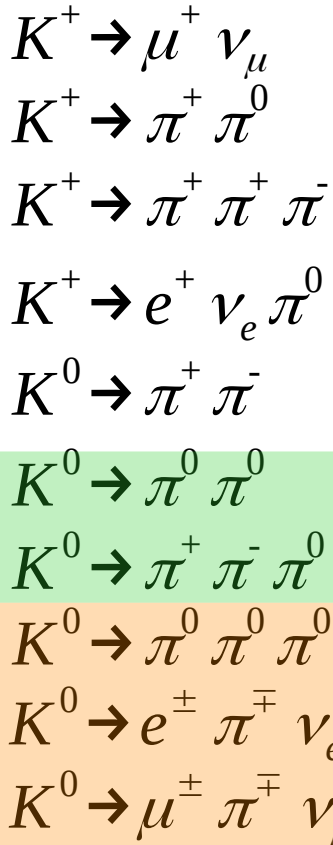
The presence of two distinct timescales in pion decay suggests two separate interactions are in play: a weaker one (slower decay) and a stronger one (faster decay)

Could isospin be at work here?
Physicists certainly applied the concept, so that $\pi^+=+1$, $\pi^0=0$, $\pi^-=-1$.

* Hideki Yukawa is said to have selected the Greek letter pi as the symbol (and name for) the pion as it resembles the Kanji character 介, "to mediate".

Pions(*) and Kaons

Kaon mass scales are $\sim 500 \text{ MeV}/c^2$ and the dominant decays are:



production of kaons occurs very quickly (sub-attosecond level) but ... always in pairs and ...

these decays happen faster ...

than these decays.

Particle	Mass (MeV/c^2)	Lifetime (s)
K^\pm	493.677	1.2×10^{-8}
K^0	497.648	Two components: 5.1×10^{-8}
		9.0×10^{-11}

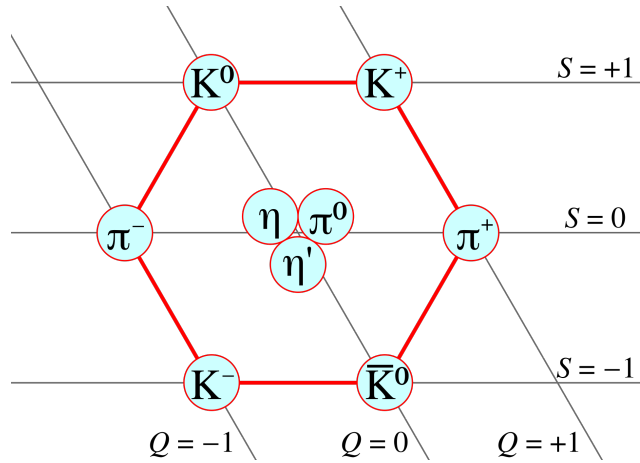


Isospin ... again?

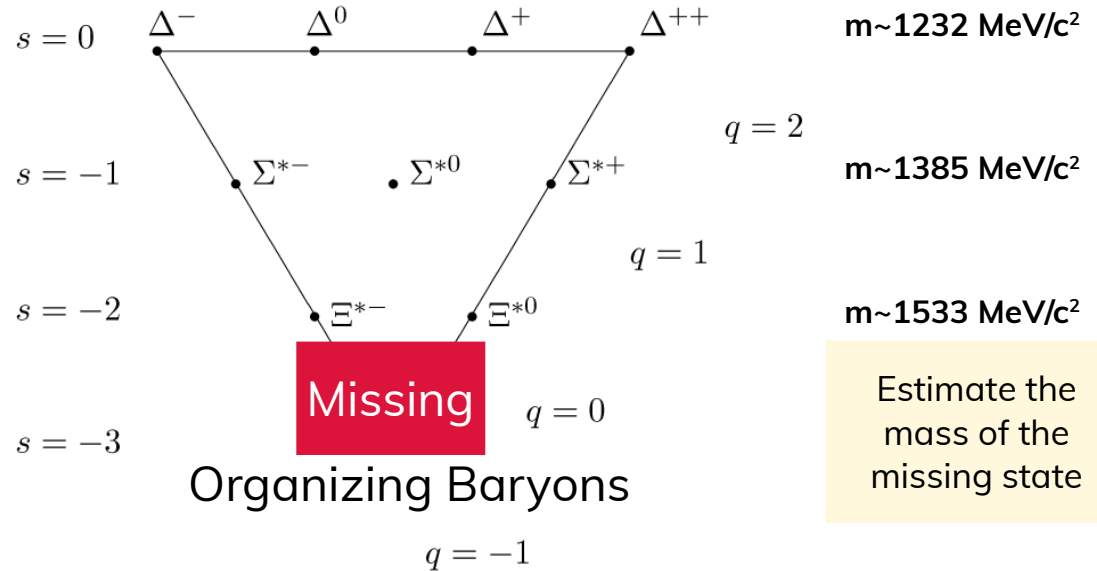
Timescale implies weak interaction controls decay

The observed fact that kaons (and other particles discovered around the same time) are always produced in pairs suggested another conserved quantity, dubbed "strangeness" (S).

Organizing Mesons and Baryons



Organizing Mesons

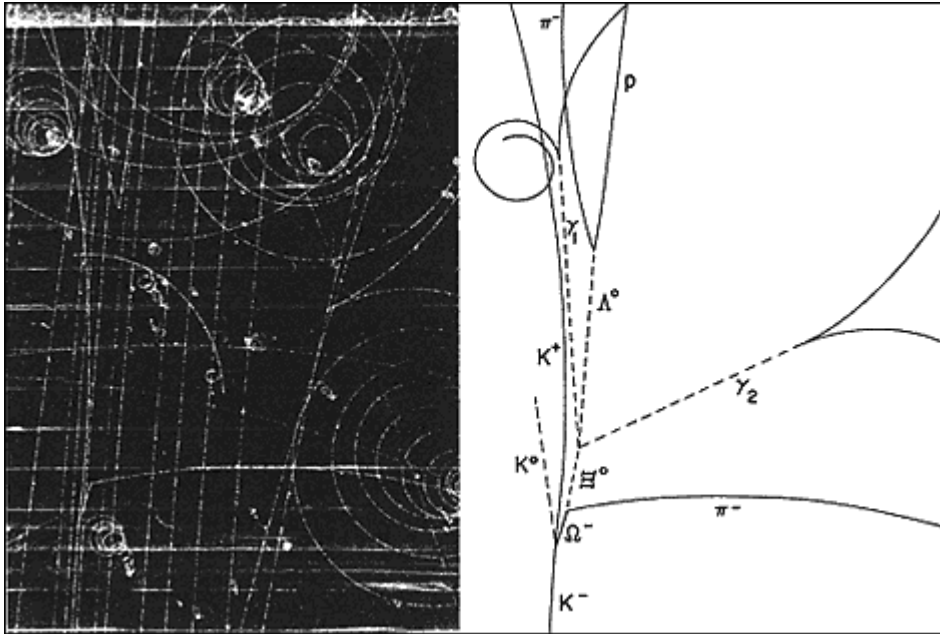


Organizing Baryons

In 1961, Murray Gell-Mann and Yuval Ne'eman independently developed an organizational scheme for the properties of then-known mesons as well as heavy particles (heavier than nucleons), the baryons.

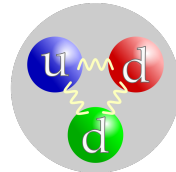
It revealed a missing piece: no particle with $q = -1$ and $s = -3$ had been found at the time this scheme was published. Gell-Mann predicted its properties, including its mass, and Gell-Mann dubbed it the Ω^- .

Organizing Mesons and Baryons

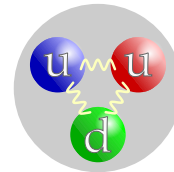


The Omega Baryon is discovered in a bubble chamber in 1964. Its mass is determined to be $1672 \text{ MeV}/c^2$.

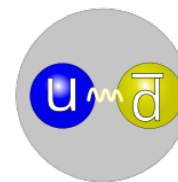
What is the underlying organizing principle that leads to all of these states of matter? There were competing ideas, but the one that won out on the strength of experimental evidence in the late 1960s and into the 1970s and 1980s was the “quark hypothesis”.



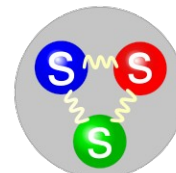
Neutron



Proton

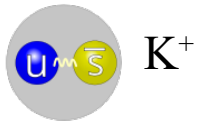


π^+



Ω^-

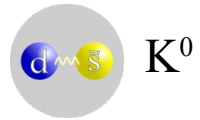
Quarks carry fractional elementary electromagnetic charge and are charged under the strong force (colour, in one of three states) and weak force (weak hypercharge).



K^+



K^-



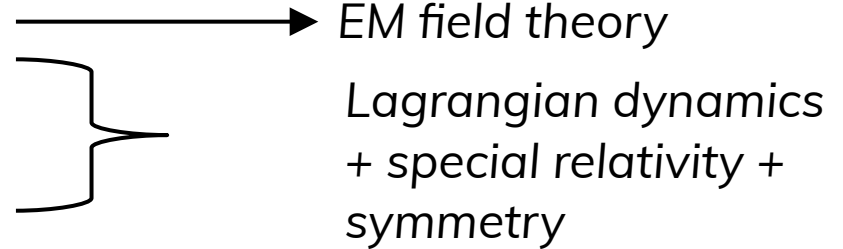
K^0



What are the Rules?

What are the rules?

- From experimental science, what are the rules of nature ca. 1965?
 - Baryon Number (B): appears to be conserved.
 - Lepton Number (L): appears to be conserved for each lepton type.
 - Electric Charge: appears to be conserved.
 - Energy: appears to be conserved.
 - Momentum: appears to be conserved.
 - Quantum mechanics: wavefunction and related concepts appear to apply to the nuclear realm as well as the atomic.



- Parity (P)

$$\Psi(x, y, z, t) \xrightarrow{P} \Psi(-x, -y, -z, t)$$

$$a \xrightarrow{P} a \quad \text{Scalars are parity-even (invariant)}$$

$$\vec{r} \xrightarrow{P} -\vec{r}$$

$$\vec{p} = m \dot{\vec{r}} \xrightarrow{P} -m \dot{\vec{r}} = -\vec{p}$$

Regular vectors are parity-odd. Check that magnitudes of position and momentum are invariant under a parity transformation

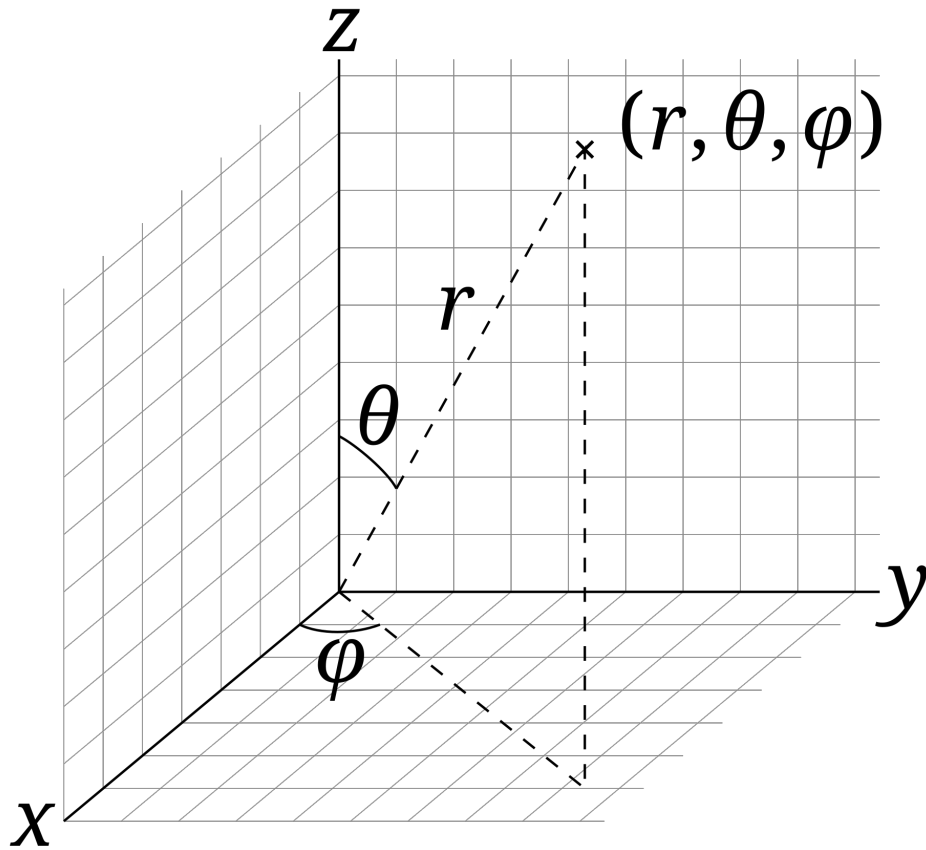
$$\vec{L} = \vec{r} \times \vec{p} \xrightarrow{P} (-\vec{r}) \times (-\vec{p}) = \vec{L}$$

All angular-momentum-like quantities are parity-even, the opposite of a regular vector → “**pseudovectors**” or “**axial vectors**”

$$\vec{a} \cdot (\vec{b} \times \vec{c}) \xrightarrow{P} (-\vec{a}) \cdot (-\vec{b} \times -\vec{c}) = -\vec{a} \cdot (\vec{b} \times \vec{c})$$

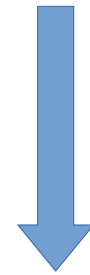
This kind of scalar is parity-odd, unlike regular scalars and is dubbed a **pseudoscalar**.

Parity Behaviour of Wave Functions



$$\Psi(\vec{r}) = R_{n\ell}(r) Y_{\ell m}(\theta, \phi)$$

↓
Parity
Even



What happens to this
under a parity
transformation?

$$\theta \xrightarrow{P} \pi - \theta$$

$$\phi \xrightarrow{P} \pi + \phi$$

$$Y_{\ell m}(\theta, \phi) \xrightarrow{P} (-1)^\ell Y_{\ell m}$$

$$\Psi(\vec{r}) \xrightarrow{P} (-1)^\ell \Psi(\vec{r})_{20}$$

Parity and Particles



- Need a convention to begin the process of labeling particles by their parity “quantum number”
 - Convention: proton, neutron, and Λ are parity even ($P=+1$)
 - From this and application of parity ideas to particle reactions (e.g., pion capture on the deuteron) one can infer the P of pions, kaons, etc.

Particle	Spin	Parity
π	0	-1
K	0	-1

Is Parity Conserved?

Particle Decay

General Idea

$A \rightarrow B + C$ (decay process)
 $P_A = P_B P_C$ (if P conserved)



What is the parity of θ^+ ?

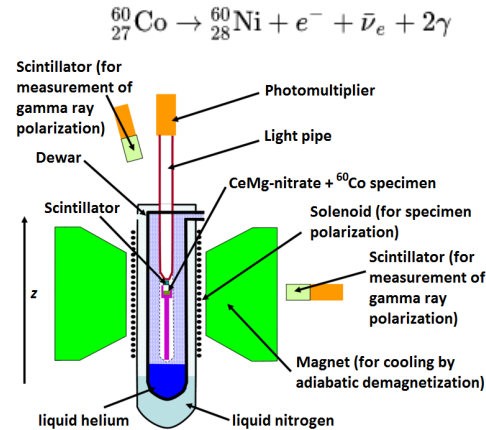


What is the parity of ζ^+ ?

It turned out the masses, lifetimes, and spins of these two initial states were the same ... they are the same particle: the K^+ .

Parity is not (always) conserved in kaon decay...

Cobalt Nucleus



$$\langle \cos(\theta_e) \rangle \approx \langle \vec{s}_{\text{Co}} \cdot \vec{p}_e \rangle = 0$$

(if P conserved)

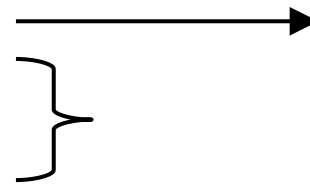
Finding: electron emission angle affected strongly by orientation of nuclear spin, photon emission relatively unaffected.



Chien-Shiung Wu 22

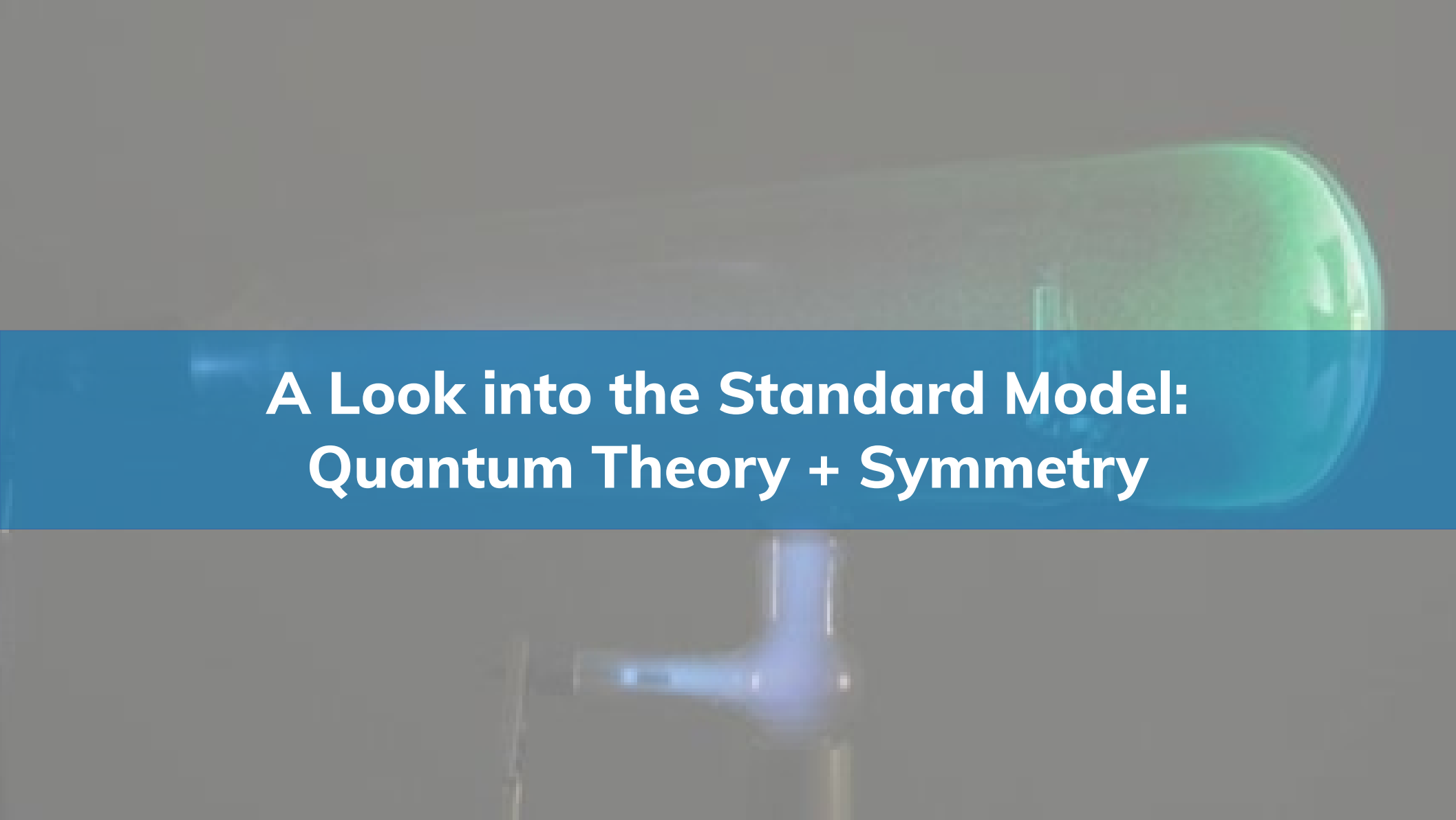
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 - Momentum: appears to be conserved.
 - Quantum mechanics: wavefunction and related concepts appear to apply to the nuclear realm as well as the atomic.
 - **Parity (P), Charge conjugation (C), and CP together are all not conserved in weak interactions but appears to be in EM and strong interactions.**



EM field theory

*Lagrangian dynamics
+ special relativity +
symmetry*



A Look into the Standard Model: Quantum Theory + Symmetry

Electromagnetism



The model needs to be “backwards compatible” with what was established in classical electrodynamics in the late 1800s. This is encoded in the Standard Model:

Lagrangian Density of the EM Field:

$$\mathcal{L}_{\text{EM Field}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \quad (\text{EM Field Tensor})$$

$$A_\mu \equiv (\phi, \vec{A}) \quad (\text{EM 4-potential})$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \left[\begin{array}{cccc} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{array} \right] \left. \vphantom{\begin{array}{c} \\ \\ \\ \end{array}} \right\} \begin{array}{l} \vec{E} = -\vec{\nabla} \phi - \frac{1}{c} \frac{\partial \vec{A}}{\partial t} \\ \vec{B} = \vec{\nabla} \times \vec{A} \end{array}$$

Maxwell's Equations are packed into this compact form and all consequences of classical EM are encoded.

Quantum Electrodynamics



We can add the Dirac equation and electromagnetic interaction between fermions (ψ) charged under electromagnetism

Lagrangian Density of QED:

$$\mathcal{L}_{\text{QED}} = \underbrace{\bar{\psi}(i \gamma^\mu \partial_\mu - m) \psi}_{\text{Kinetic Energy and Mass of } \psi} - \underbrace{e \bar{\psi} \gamma^\mu Q \psi A_\mu}_{\psi \text{ and EM 4-potential interaction}} - \underbrace{\frac{1}{4} F_{\mu\nu} F^{\mu\nu}}_{\text{Kinetic energy of the EM 4-potential}}$$

Q is the charge operator, whose eigenvalue is -1 for an electron.

e is the elementary charge

γ_μ are the Dirac gamma matrices, which are a higher-dimensional representation of the Pauli spin matrices.

Classical electrodynamics is recoverable from this (e.g., electrons in an electromagnetic field)

Electroweak Theory



The Standard Model contains a single description of the electromagnetic and weak interactions together; these are understood as being two aspects of a single interaction that manifests at higher energy.

Lagrangian Density of EW theory:

$$\mathcal{L}_{EW} = \bar{\psi}_L \gamma^\mu (i\partial_\mu - g \frac{1}{2} \vec{\tau} \cdot \vec{W}_\mu - g' \frac{Y}{2} B_\mu) \psi_L + \bar{\psi}_R \gamma^\mu (i\partial_\mu - g' \frac{Y}{2} B_\mu) \psi_R - \frac{1}{4} \vec{W}_{\mu\nu} \cdot \vec{W}^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \text{mass terms}$$

Fermion kinetic energies and interactions through the γ , W^\pm , and Z^0 .
Only left-handed fermions can interact via the W bosons (parity violation).

For example, a massless fermion is left-handed ($\text{sgn}(\mathbf{s} \cdot \mathbf{p}) = -1$) and a massless fermion is right-handed ($\text{sgn}(\mathbf{s} \cdot \mathbf{p}) = +1$)

Kinetic energy of the γ , W^\pm , and Z^0 and their self-interactions.

Wait for it ...

Parity violation in the weak interaction, etc. QED is included in this part of the SM Lagrangian density.

Quantum Chromodynamics



The Standard Model encodes the strong interaction with a piece of the Lagrangian density that is specific to the quarks, since they are the only particles that experience the strong interaction. The theory of colour charge (three states: red, green, blue; plus anti-colours: anti-red, anti-green, anti-blue) and quarks is “Quantum Chromodynamics” (QCD):

Lagrangian Density of QCD theory:

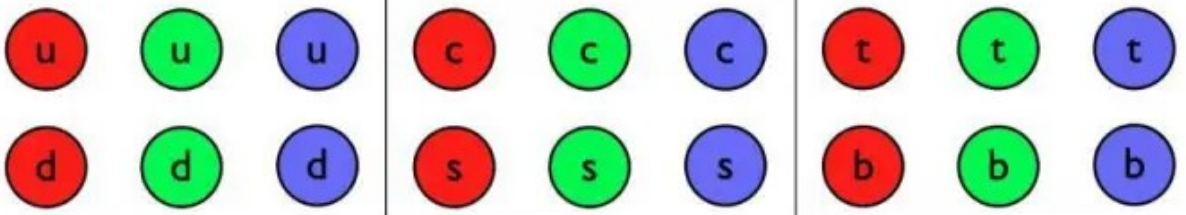

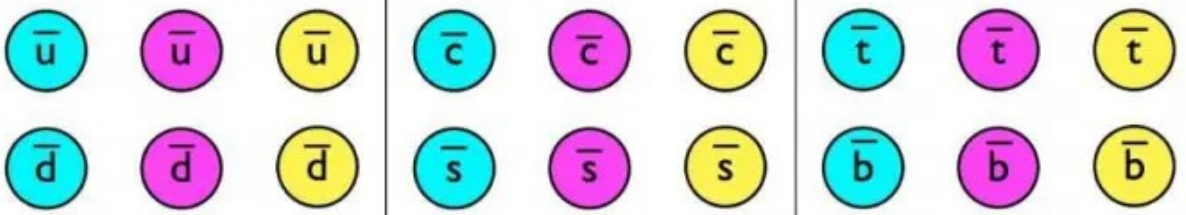
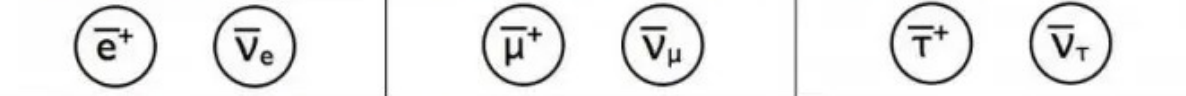

$$\mathcal{L}_{\text{QCD}} = \underbrace{i\bar{U}(\partial_\mu - ig_s G_\mu^a T^a)\gamma^\mu U}_{\text{Interactions between up-type quarks (up, charm, top) as well as kinetic energy of those quarks.}} + \underbrace{i\bar{D}(\partial_\mu - ig_s G_\mu^a T^a)\gamma^\mu D}_{\text{Interactions between down-type quarks (up, charm, top) as well as kinetic energy of those quarks.}} - \underbrace{\frac{1}{4}G_{\mu\nu}G^{\mu\nu}}_{\text{Gluon field kinetic energy, including self-interaction (gluons carry colour charge)}}$$

The strong interaction as described here captures the key elements: quarks are never free from being bound by gluons, gluons self-interact.

Gluons carry bare colour states (8 possible):

$$\begin{array}{ll} (r\bar{b} + b\bar{r})/\sqrt{2} & -i(r\bar{b} - b\bar{r})/\sqrt{2} \\ (r\bar{g} + g\bar{r})/\sqrt{2} & -i(r\bar{g} - g\bar{r})/\sqrt{2} \\ (b\bar{g} + g\bar{b})/\sqrt{2} & -i(b\bar{g} - g\bar{b})/\sqrt{2} \\ (r\bar{r} - b\bar{b})/\sqrt{2} & (r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6}. \end{array}$$

Colour Charge: Fermions and Bosons

	Quarks
	Leptons
	Anti-Quarks
	Anti-Leptons
	Bosons

Each colour state is a distinct state of a fermion or gluon. The up quark has 3 possible distinct colour states. The electron has none (colourless). The gluon has 8, and all other known bosons have none.

The Standard Model

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{EW}} + \mathcal{L}_{\text{QCD}}$$

Taking Stock of Symmetries

Built into the Standard Model are a number of Gauge Symmetries – transformations of the fields that leave something invariant.

$$g' Y_W B_\mu$$



The U(1) symmetry in this part of the Lagrangian implies that interactions conserve electroweak current (weak hypercharge) under Z and γ interactions.

Symmetric under the group properties of U(1), the group of unitary transformations in 1 dimension. Example: the Dirac Equation

$$\mathcal{L} = i \bar{\psi} \gamma_\mu \partial^\mu \psi - m \bar{\psi} \psi$$

How does ψ transform under U(1)? $\psi \rightarrow e^{i\alpha} \psi$ **α is a real constant**

Determine what is required for this condition to be true... $\partial_\mu \psi \rightarrow e^{i\alpha} \partial_\mu \psi$
 $\bar{\psi} \rightarrow e^{-i\alpha} \bar{\psi}$

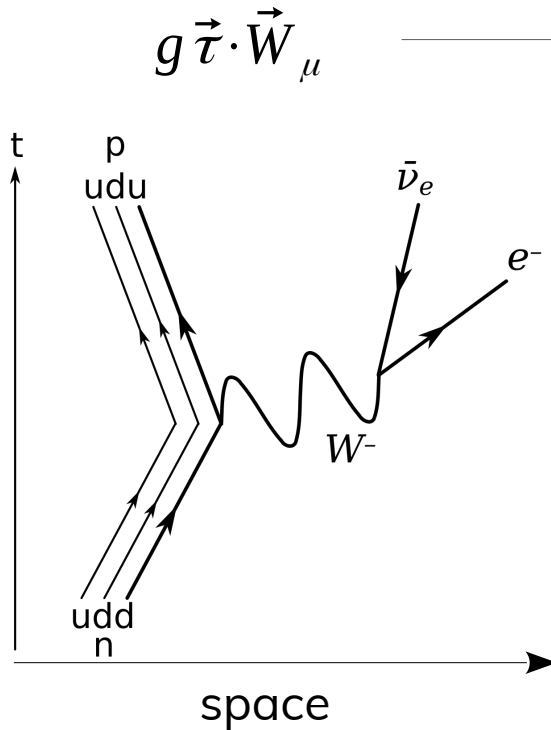
$$\delta \mathcal{L} = 0$$



This condition leads to the requirement that charge must be conserved in order to satisfy the requirement of invariance under this transformation \rightarrow in other words, if the equation is invariant under a U(1) transformation, the consequence is that charge is conserved.

Taking Stock of Symmetries

Built into the Standard Model are a number of Gauge Symmetries – transformations of the fields that leave something invariant.



$$g \vec{\tau} \cdot \vec{W}_\mu$$

→ Symmetric under the group properties of SU(2), the group of special unitary transformations in 2 dimensions, corresponding to rotations in weak isospin space → implies left-handed interactions can transform fermions (e.g., electrons into electron-neutrinos and vice versa) and that this interaction makes no distinction between flavours of fermions.

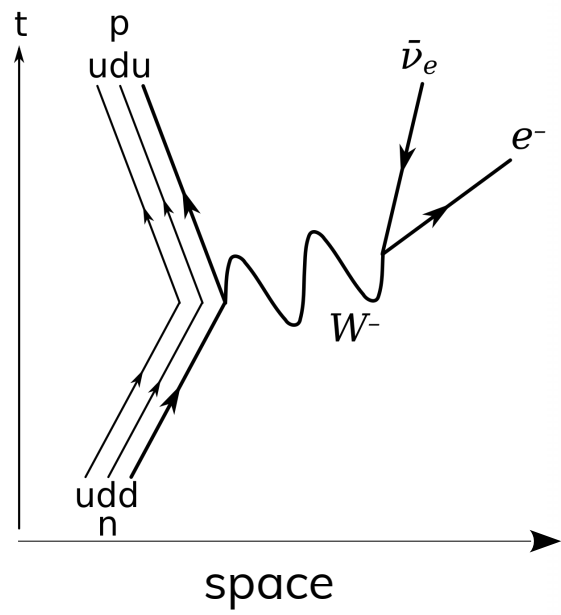
The W boson interaction is “universal” with all left-handed fermions → to first approximation, it cannot “tell the difference” between leptons and quarks.

Example: $W^+ \rightarrow u\bar{d}, c\bar{s}, e\nu_e, \mu\nu_\mu, \tau\nu_\tau$ are all the same from the perspective of the W boson. [$W \rightarrow t\bar{b}$ is suppressed by conservation of energy/kinematics ($m_t > m_W$)]

Feynman Diagrams

Feynman diagrams are mistaken for cartoons but are, in fact, graphical representations of quantum field theory calculation elements (terms in an expansion of a scattering amplitude, each a representation of an order of an expansion of the amplitude using perturbation theory).

Diagram elements are assembled respecting conservation rules and represent underlying mathematical guidelines.



QED rules excerpted from Halzen & Martin, "Quarks and Leptons", pg. 149

TABLE 6.2
Feynman Rules for $-i\mathcal{R}$

		Multiplicative Factor
● External Lines	Spin 0 boson (or antiboson)	1
	Spin 1/2 fermion (in, out)	u, \bar{u}
	antifermion (in, out)	\bar{v}, v
	Spin 1 photon (in, out)	$\epsilon_\mu, \epsilon_\mu^*$
● Internal Lines—Propagators (need $+i\epsilon$ prescription)	Spin 0 boson	$\frac{i}{p^2 - m^2}$
	Spin 1/2 fermion	$\frac{i(\not{p} + m)}{p^2 - m^2}$
	Massive spin 1 boson	$\frac{-i(g_{\mu\nu} - p_\mu p_\nu / M^2)}{p^2 - M^2}$
	Massless spin 1 photon (Feynman gauge)	$\frac{-ig_{\mu\nu}}{p^2}$
	● Vertex Factors	Photon—spin 0 (charge $-e$)
Photon—spin 1/2 (charge $-e$)		$ie\gamma^\mu$

Loops: $\int d^4k / (2\pi)^4$ over loop momentum; include -1 if fermion loop and take the trace of associated γ -matrices
 Identical Fermions: -1 between diagrams which differ only in $e^- \leftrightarrow e^-$ or initial $e^- \leftrightarrow$ final e^+

Interactive Exercise:

Tau Lepton Decay to Pion(s)

The tau lepton (a fermion of mass $1777 \text{ MeV}/c^2$) decays to states involving one or more pion and one or more neutrinos. Sketch some examples of this process using the following rules:

- Electric charge is conserved, colour charge is conserved
- Energy and momentum are conserved at each “vertex” of a Feynman diagram (which means also from initial to final states). Mind also the spin angular momentum (pions are scalars, meaning spin-0).
- A meson (e.g., π) is a bound state of two quarks (bound through the strong interaction).
- Remember that $E = mc^2$. If there is sufficient energy available, quantum mechanics can pair-produce particles from a field!

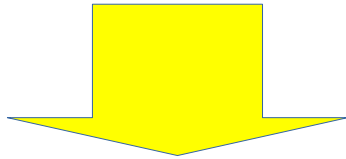
π^+	$u \bar{d}$
π^0	$u \bar{u}$ or $d \bar{d}$
π^-	$\bar{u} d$



Consequences of the Standard Model (Glashow-Weinberg-Salam Theory)

Predictive Power: Masses of the Z and W bosons

$$G_F = \frac{\pi \alpha}{\sqrt{2} \sin^2 \theta_W M_W^2}$$



Fermi's constant (G_F) is associated to the strength of the weak interaction and was measured in the 1970s from beta decay (e.g., muon decay!). In 1975, $G_F \sim 1.02 \times 10^{-5} \text{ m}_p^{-2}$. The CDHS neutrino scattering experiment constrained the $\sin^2 \theta_W$ to be 0.24 (10% uncertainty) in 1977. $\alpha = 1/137$.

$$M_W^2 = \frac{\pi \alpha}{\sqrt{2} \sin^2 \theta_W G_F} \rightarrow M_W \approx 80 \text{ GeV}/c^2$$

$$M_Z = M_W / \cos \theta_W \approx 90 \text{ GeV}$$

Predictive Power: Decay Rates of the W boson and Tau Lepton from universality



- Assume a real (as opposed to virtual) W boson with a mass of 80 GeV. Assuming universal couplings to all flavour doublets (e.g., ν_e and e^+ , or u and d), what will be the:
 - branching fraction of $W \rightarrow$ leptons
- Tau lepton decay proceeds through W boson emission (weak decay), but the W boson is virtual ($m \ll 80$ GeV). Again using universality, predict the branching fraction of the tau to leptons. **HINT: of the possible W decays to quark doublets, and recalling the tau lepton decay lesson, are both ud and cs allowed?**
- Compare your calculations to measured values in the Particle Data Listings (pdglive.lbl.gov)

A “branching fraction” to a given final state, S , or collection of final states is the ratio of the rate of decay to S divided by the rate of all possible decays.

This Way to Gold

- The Standard Model has been an extraordinary tool for discovery, both in how it developed and how it was used.

1979. Glashow (US), Salam (Pakistan), Weinberg (US)

1980. Cronin (US), Fitch (US).

1984. Rubbia (Italy), van der Meer (Netherlands)

1988. Lederman (US), Schwartz (US), Steinberger (US)

1990. Friedman (US), Kendall (US), Taylor (US)

1995. Perl (US), Reines (US)

1999. t'Hooft (Netherlands), Veltman (Netherlands)

2004. Gross (US), Politzer (US), Wilczek (US)

2008. Kobayashi (Japan), Maskawa (Japan), Nambu (US)

2013. Englert (Belgium), Higgs (UK)



Summary and Next Steps



- The Standard Model has extreme predictive power. This is what has made it so useful.
 - Once you nail down some of the free parameters (it has 19, ignoring neutrino mixing ... save that for next lecture!), you can use relationships to constrain/predict others.
- We'll explore briefly the mechanism by which mass (fermion and boson) arises in the Standard Model.
- The main focus of the last lecture is on places where the Standard Model is incomplete or just broken. This sets the stage for the challenge to describe the universe.



APPENDIX

References

- S. Navas et al. (Particle Data Group), to be published in Phys. Rev. D 110, 030001 (2024)
- D. C. Cassidy. “Beyond Uncertainty: Heisenberg, Quantum Physics, and the Bomb”. Bellevue Literary Press. 2009.