# TRISEP 2024 - The Standard Model
# Class Worksheet

Stephen Sekula

## 1 Day 1

If you need to look up numbers (e.g., masses, fundamental constants) please use the Particle Data Group website (pdg.lbl.gov). Their interactive tables are an excellent source of data (pdglive.lbl.gov).

### 1.1 Estimating Nuclear Energy Scales

Nuclear processes often emit photons, which we would associate with the movement of electric charge within the nucleus. We wish to estimate the energy scale of nuclear interactions using a purely classical calculation. Imagine a tritium nucleus (one proton and two neutrons) into which you then bring a proton to form a helium nucleus. Assume you are bringing the proton from an infinite distance away and placing it so that the centres of the two protons are 1 fm apart.

- Estimate the work required to assemble these charges in this way. *This provides a very rough classical estimate of the energy scale associated with the nucleus (e.g., the potential energy stored in the electromagnetic field, which could be converted into other forms of energy).*
- **BONUS:** estimate the wavelength of a photon that could be released from a nuclear process after calculating the above work (here, you are free to use modern physics). What class of photon is this? *(radio, infrared, visible, uv, x-ray, gamma ray)*

### 1.2 Converting mass units

Using Einstein's relation between mass, energy, and momentum, compute the rest mass of the electron in units of MeV/c$^2$ starting from the MKS value, $m_e = 9.109 \times 10^{-31}$ kg.

### 1.3 Beta Decay of Free Neutron

Consider a free neutron at rest. Beta decay is the emission of a fast-moving electron from a nuclear process. Let us hypothesize that the neutron decays as $n^0 \rightarrow p^+ e^-$ (this conserves electric charge and was a reasonable first guess for this process in the early 1900s). What will be the kinetic energy of the electron? You should use relativistic mechanics to do this calculation.

### 1.4 Muon Lifetime and Survival

The famous "Mount Washington" or "Frisch-Smith" experiment used an apparatus to observe cosmic ray muons at high altitude (atop Mt. Washington). The experiment observed 563 muons per hour at an altitude of 1917 m above sea level. The same experiment measured 412 muons per hour at sea level. The experiment was designed to sample muons with velocities between 0.9950c and 0.9954c. The mean lifetime of the muon measured at rest with respect to the muon is 2.1969811 $\mu$s.

- Using a perfectly classical calculation (ignore special relativity), how many muons SHOULD the experiment have measured at sea level?

- Now using special relativity, estimate the number of muons the experiment was expected to measure at sea level.

- Compare the latter calculation to the observed numbers. Comment on any uncertainties based on information given in the problem.

## 1.5 Fitting the Muon Lifetime

A data set from a muon decay experiment is available online. This is real data from the instrument depicted in the slides and operated for many years at Southern Methodist University in Dallas, TX. The data represents continuous data taking from about 4 months in 2020.

Download this data and, using your favourite software framework (Excel, Python, C++, etc.) determine the observed lifetime of the muon assuming that the data can be described by a simple model involving both real muon decay coincidences (a stopping muon energy pulse followed by the muon decay energy pulse) and random noise coincidences in the instrument (which should be uniformly distributed in time),

$$P(t) = Ae^{-t/\tau} + B.$$

- Determine the three parameters - A, B, and $\tau$- by fitting the model to the data (using a least-squares, maximum likelihood, etc. approach).

- Compare your result for $\tau$ to the accepted value, $2.1969811\,\mu$s. Comment on your observation(s).

**Format of the Data and Important Comments** The data is formatted into a file with two columns. The first column is the time between the two pulses in nanoseconds. The second column is the timestamp of the event in UNIX time. You should not need the second column.

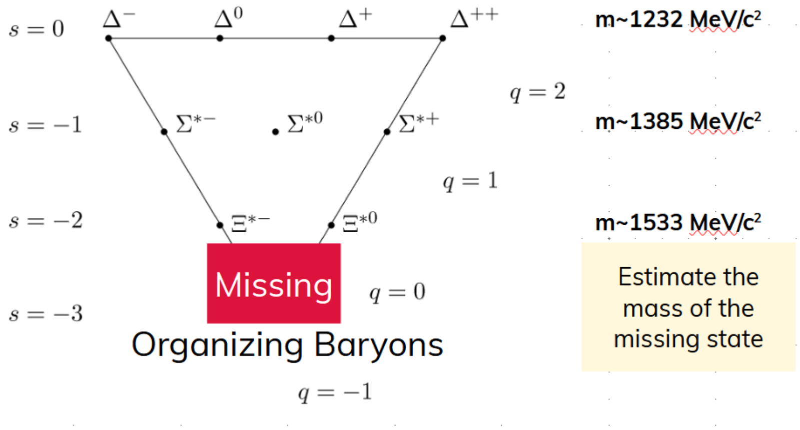
The maximum time between pulses in this instrument is 20000 ns. If the experiment counts to 20000 ns after the initial (trigger) pulse, it times out and stores that timeout as a muon decay candidate with exactly 20000 ns of lifetime. This is unphysical. ***However, timeout events have already been eliminated from the dataset. You should use data from 1000-20000 in your fit.***

Estimate

# 2 Day 2

## 2.1 Estimate the Mass of the Omega Baryon

After organizing baryons (which later came to be understood as matter states built from 3 quarks), Murray Gell-Mann and Yuval Ne'eman noted that there seemed to be a missing particle in what was known as the "Baryon Decuplet" - a 10th, but undiscovered, particle appeared to be needed to complete the picture. Gell-Mann used a simple method to estimate the mass of the missing state, which he dubbed the "Omega Baryon". Use the information below and develop a simple method to estimate the mass of this 10th state.



## 2.2 Estimate the branching fractions of a W boson, and a tau lepton, to leptonic final states

A *branching fraction* is the number of ways a particle can decay to a final state (or set of final states) divided by the number of all possible decays. It is also known as a *branching ratio*. You could write this, in terms of state-counting, as

$$\mathcal{B} \equiv \frac{\sum_{i\subset all} N_i}{\sum_{all} N_j}.$$

For the numerator and denominator, it's important to remember that your goal is to count all possible ways to arrange a system into a configuration, including ways the same apparent final state could be arranged so that it's otherwise identical to the observer. *(he said hintingly)*

Treat the W boson couplings to the particles in each Standard Model "doublet" as equal, which is a consequence of the symmetry of the weak sector of the Standard Model Lagrangian density. That means that the following ways of decaying are all expected to be equally likely (up to kinematic effects, e.g. some particles are heavier than others and this comes at a cost, but not in the coupling):

$$W^+ \to u\bar{d}, c\bar{s}, e^+\nu_e, \mu^+\nu_\mu, \tau^+\nu_\tau.$$

The mass of the W boson is about 80 GeV/c$^2$and the mass of the top quark is 173 GeV/c$^2$, so a real W (with the nominal mass) cannot decay to a real top quark (with its nominal mass). We will ignore effects like virtual particles and quark flavour mixing for this problem. Keep in mind that quarks and leptons have electric charge, and quarks also have colour charge (red, green, and blue; anti-red, anti-green, and anti-blue), and the W boson is electrically charged but colourless. Overall, energy and momentum must be from the initial state to the final state.

- Estimate the branching fraction of a real W to real leptons.
- I was just kidding about that "ignoring virtual particles" thing. Now compute the tau lepton's branching fractions to leptonic final states, knowing that the tau decay proceeds entirely through the W

boson. ($\tau^+ \to W^{+*}\bar{\nu}_\tau$). Keep in mind that the tau cannot decay to anything heavier than itself, and that quarks must ultimately form bound states (e.g., mesons, bound by gluons) in the final state.

| Particle | Mass [GeV/c$^2$] | Electric Charge [e] | Colour Charges |
|---|---|---|---|
| $W^+$ | 80 | +1 | NONE |
| u | ~0.0023 | +2/3 | r,g,b |
| d | ~0.0048 | -1/3 | r,g,b |
| c | 1.275 | +2/3 | r,g,b |
| s | 0.095 | -1/3 | r,g,b |
| t | 173 | +2/3 | r,g,b |
| b | 4.2 | -1/3 | r,g,b |
| $e^+$ | 0.000511 | +1 | NONE |
| $\nu_e$ | Negligible | 0 | NONE |
| $\mu^+$ | 0.106 | +1 | NONE |
| $\nu_\mu$ | Negligible | 0 | NONE |
| $\tau^+$ | 1.78 | +1 | NONE |
| $\nu_\tau$ | Negligible | 0 | NONE |