Dark Matter and Cosmology II: Advanced Dark Arts and Alchemy

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First, some discussion on dimensional analysis as a way to understand the essentials of anything.

$$\Delta X \Delta p \geq \frac{\hbar}{2}$$

$$\Delta E \Delta T \geq \frac{\hbar}{2}$$

$$\lambda_c = \frac{2\pi\hbar}{m}$$

$$\lambda_d = \frac{2\pi\hbar}{p}$$

Set $\hbar$=c=1

$$G = \frac{1}{M_{PL}^2} = \frac{8\pi}{m_{pl}^2} = 10^{-38}\ \mathrm{GeV}^{-2}$$

$$\mathrm{GeV} = \frac{1}{2 \times 10^{-14}\ \mathrm{cm}}$$

$$\mathrm{GeV} = \frac{1}{7 \times 10^{-25}\ \mathrm{s}}$$

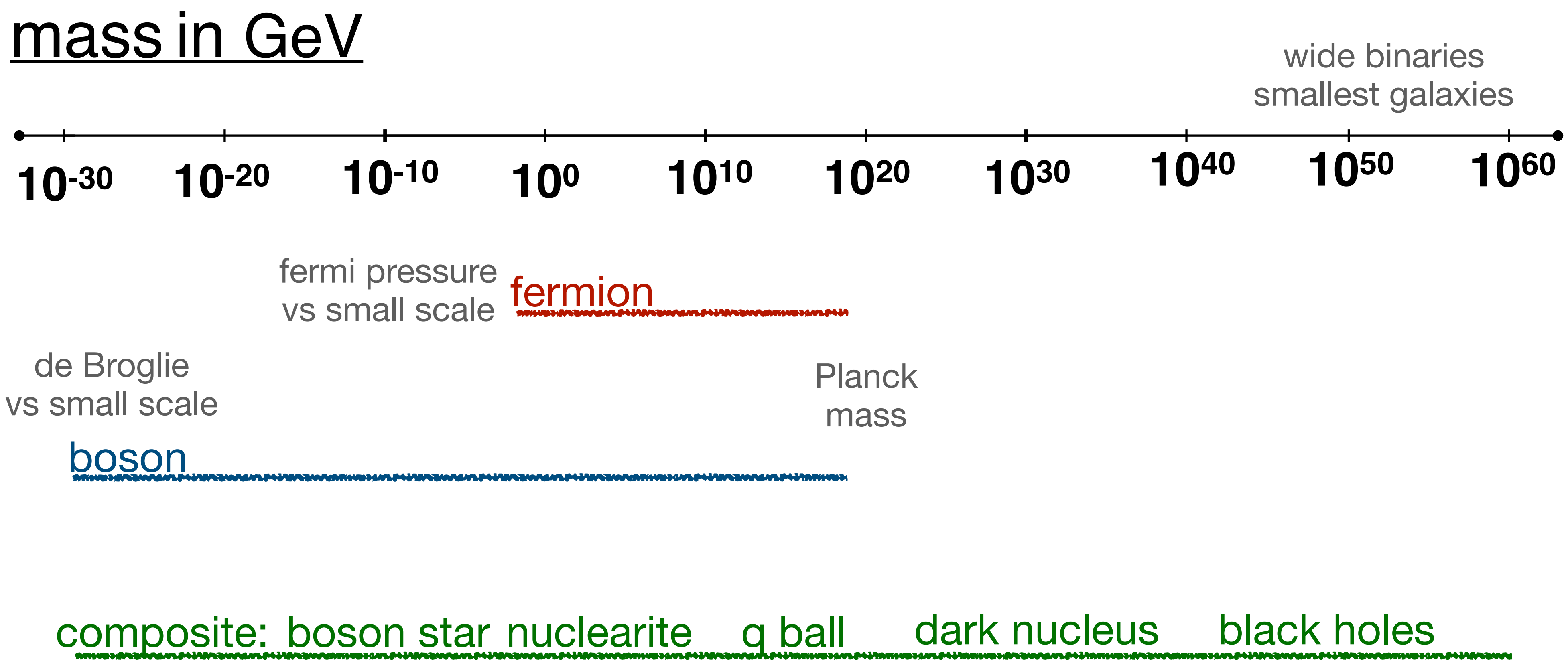
This is also called natural units.

$$[E] \sim \left[\frac{1}{X}\right] \sim \left[\frac{1}{T}\right]$$

$$[X] \sim [T] \sim \left[\frac{1}{E}\right]$$

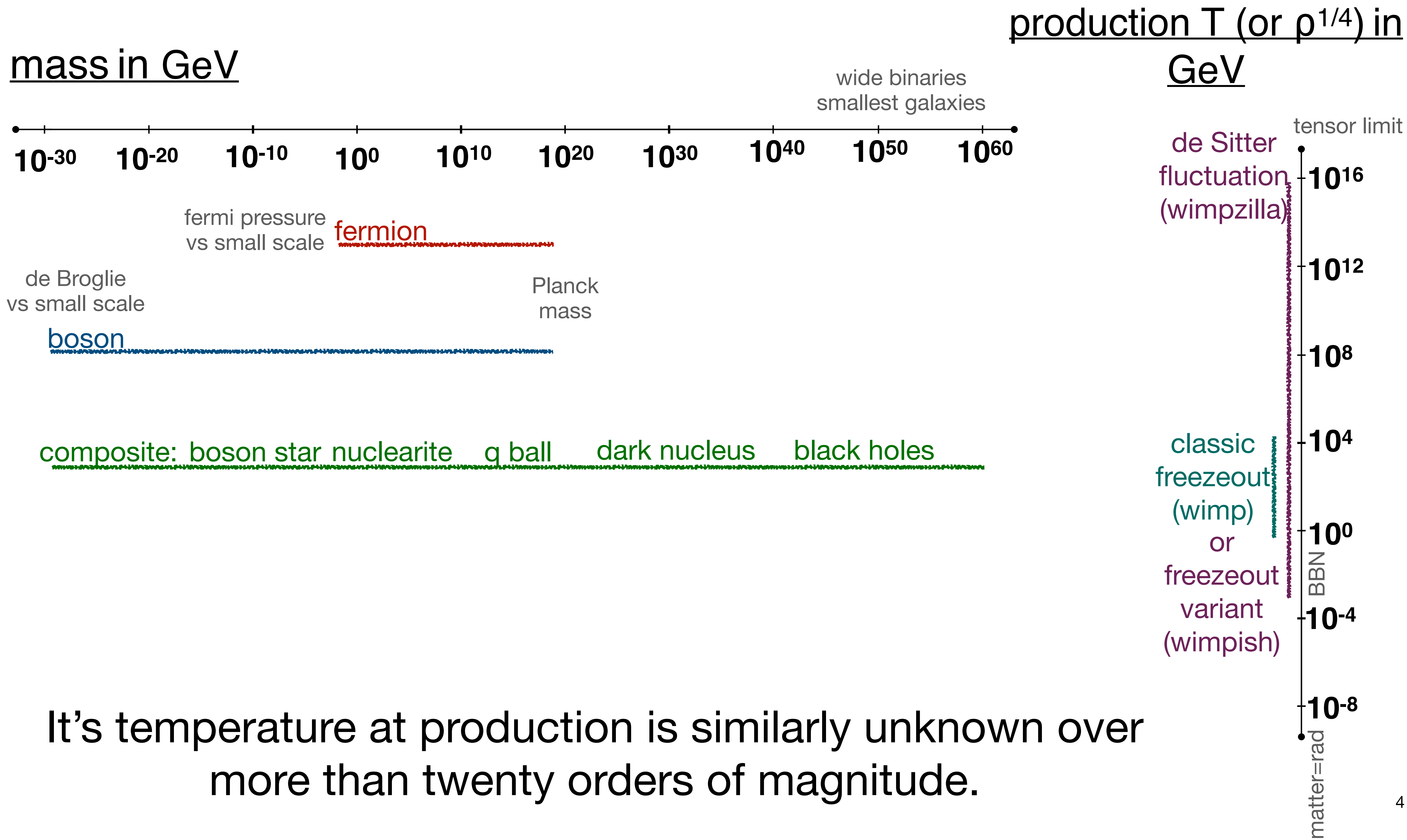
proton mass ~GeV

What do we know about dark matter?



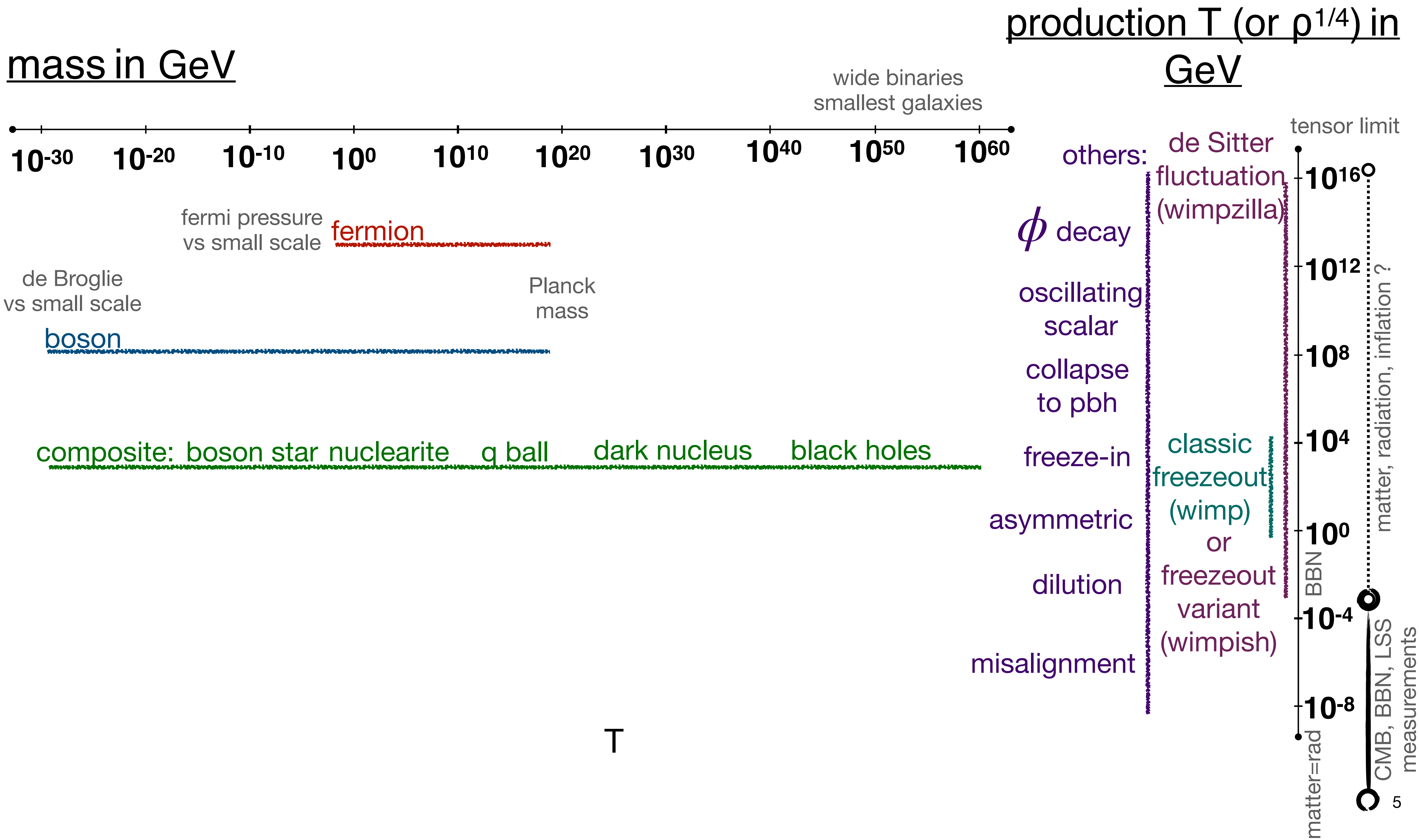
Its mass falls along a wide range, for fermions, bosons, or composite dark matter the allowed masses are different.

What do we know about dark matter?

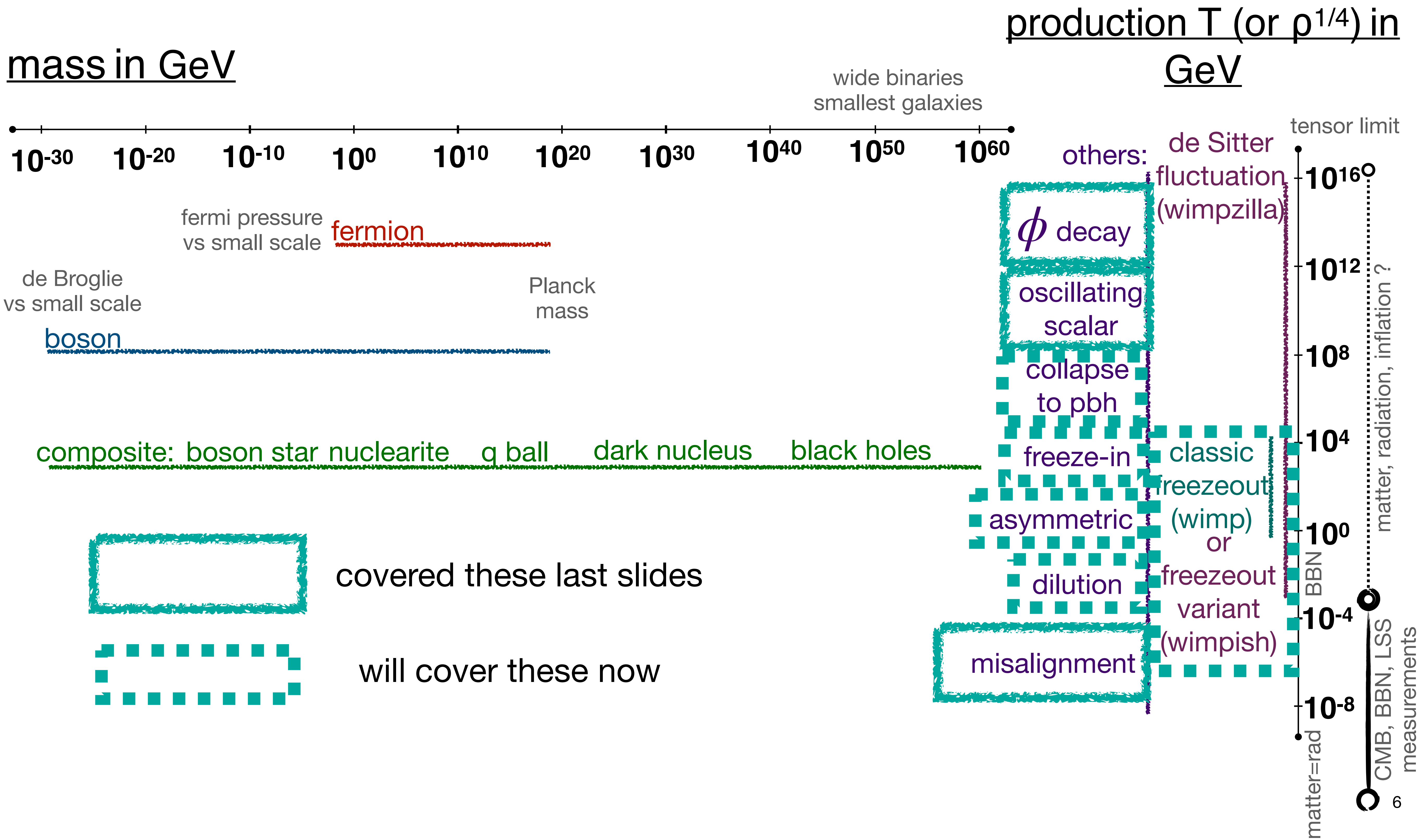


It's temperature at production is similarly unknown over more than twenty orders of magnitude.

What do we know about dark matter?



What do we know about dark matter?

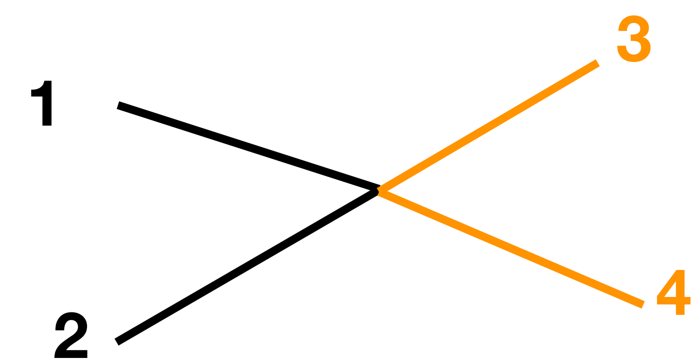


Boltzmann equations

The full form of the Boltzmann equation accounts for the dynamics of particles in a thermal bath, beginning with the Liouville operator for the phase space density f(p,t) in an FRW background

$$\mathbf{L}[f] = \frac{\partial f}{\partial t} - \frac{\dot{a}}{a} p \frac{\partial f}{\partial p}$$

In the absence of interactions, L = 0,
with 2<->2 interactions (in the com frame),



$$\mathbf{L}[f] = \mathbf{C}[f]$$

$$\mathbf{C}[f(p_1)] = -\frac{1}{E_1} \int \prod_{i=2}^{4} dP_i \, (2\pi)^4 \, \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \Big(f_1 f_2 (1 \pm f_3)(1 \pm f_4) |\mathscr{M}(12 \to 34)|^2$$
$$- f_3 f_4 (1 \pm f_1)(1 \pm f_2) |\mathscr{M}(34 \to 12)|^2 \Big)$$

$$dP_i = \frac{d^3 p_i}{(2\pi)^3 2E_i}$$

Where M are the amplitudes for forward and reverse interactions, the ± applies for bosons/fermions. Ignoring bose/fermi statistics, the general solution for the f's will have the form

$$f_{\text{eq}} = e^{\mu(t) - \beta(t) E}$$

Where μ is the chemical potential and $\beta = T^{-1}$ is the inverse temperature

In the limit m >> T, $\beta E \to 0$, and in the limit T>>m, $\mu \to 0$

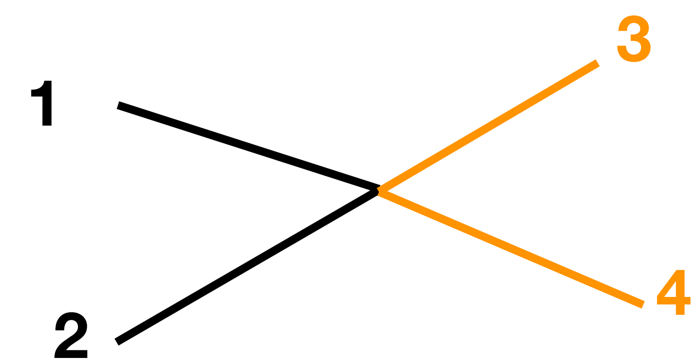
Boltzmann equations

We will want to re-express the phase space density in terms of number densities

$$n(t) = g\int \frac{d^3p}{(2\pi)^3} f(p,t)$$

Here g counts the spin states, the same as we saw in the relativistic bath d.o.f.

This leads to the more standard Boltzmann eq. form



$$\dot{n} + 3Hn = \int \frac{d^3p_1}{(2\pi)^3}\mathbf{C}[f(p_1)]$$

$$\dot{n} + 3Hn = -\int \prod_{i=1}^{4} dP_i\,(2\pi)^4\,\delta^{(4)}(p_1+p_2-p_3-p_4)\left(f_1 f_2|\mathcal{M}_{12,34}|^2 - f_3^{eq} f_4^{eq}|\mathcal{M}_{34,12}|^2\right)$$

Where in the last step, we have assumed that 1 is identical to 2, and 3,4 are “equilibrium” relativistic SM thermal bath particles, we next re-express this with cross-sections using

$$\int \prod_{i=3}^{4} dP_i\,(2\pi)^4\,\delta^{(4)}(p_1+p_2-p_3-p_4)|\mathcal{M}_{12,34}|^2 = 4\sqrt{(p_1.p_2)^2-(m_1m_2)^2}\,\sigma_{12,34}$$

Which leads to the Boltzmann equation in terms of cross-sections defined in the com frame for the fields, using the Moller velocity

$$(v_{\mathrm{Moll}})_{ij} = \frac{\sqrt{(p_i.p_j)^2-(m_im_j)^2}}{E_iE_j} = \sqrt{\left|\frac{\vec{p}_i}{E_i}-\frac{\vec{p}_j}{E_j}\right|^2 - \left|\frac{\vec{p}_i}{E_i}\times\frac{\vec{p}_i}{E_i}\right|^2}$$

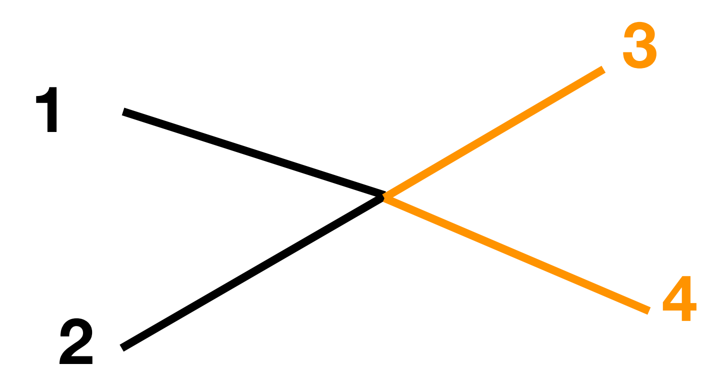
$$\boxed{\dot{n} + 3Hn = -\int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3} f_1 f_2\,(\sigma v_{\mathrm{Moll}})_{12\to34} + \int \frac{d^3p_3}{(2\pi)^3}\frac{d^3p_4}{(2\pi)^3} f_3^{eq} f_4^{eq}\,(\sigma v_{\mathrm{Moll}})_{34\to12}}$$

Boltzmann equations

The main point of these slides for our purposes, is to formally define the velocity-averaged scattering cross-section, which appears a lot in simplified Boltzmann equation treatments:

$$\boxed{\dot{n}+3Hn=-\int\frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}f_1f_2\left(\sigma v_{\mathrm{Moll}}\right)_{12\to34}+\int\frac{d^3p_3}{(2\pi)^3}\frac{d^3p_4}{(2\pi)^3}f_3^{\mathrm{eq}}f_4^{\mathrm{eq}}\left(\sigma v_{\mathrm{Moll}}\right)_{34\to12}}$$

$$\langle\sigma v\rangle=\frac{\int\frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}f_1^{\mathrm{eq}}f_2^{\mathrm{eq}}\left(\sigma v_{\mathrm{Moll}}\right)}{\int\frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}f_1^{\mathrm{eq}}f_2^{\mathrm{eq}}}$$



Using our definition of the number in terms of phase space density and assuming there is equity between 12->34 and 34->12 processes, we arrive at the abbreviated Boltzmann equation shown in the literature

$$\dot{n}+3Hn=\langle\sigma v\rangle\left(n_{\mathrm{eq}}^2-n^2\right)$$

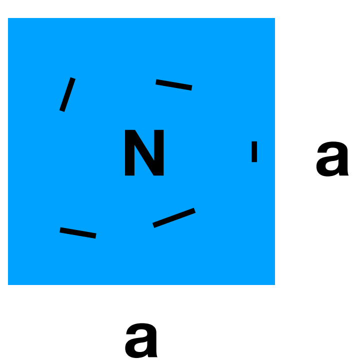
Note that here, n is going to be some dark sector particle we're interesting in tracking the evolution of, while n_{eq} is the equilibrated particles, which annihilate to produce n

Boltzmann II: Electric Bugaloo

Start with the number density of particles, decreasing in an expanding box of length a.

$$\frac{dN}{dt} = 0$$

n = N/a^3



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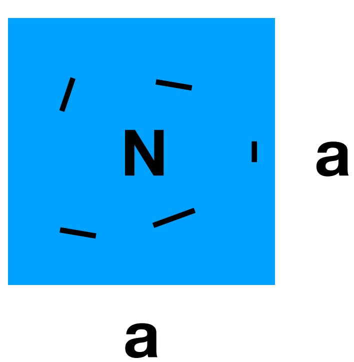


$$\frac{dN}{dt} = 0$$

$$\frac{d(na^3)}{dt} = 0$$

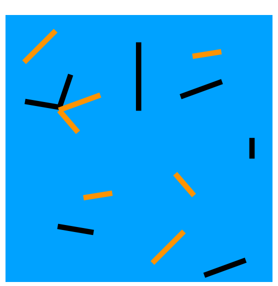
$$a^3 \frac{dn}{dt} + 3na^2\dot{a} = 0 \qquad\qquad H \equiv \frac{\dot{a}}{a}$$

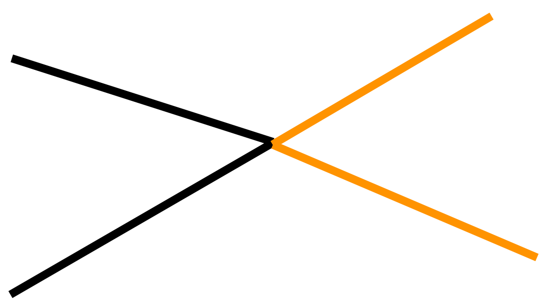
$$\frac{dn}{dt} + 3Hn = 0$$

This just follows from the assumption that total particle number is conserved (we assume otherwise soon).

Now we consider how the number density of particles changes if dark matter can annihilate to other particles,

annihilation rate per unit volume: $n^2 \langle \sigma_a v \rangle$





$$\frac{dn}{dt} + 3Hn = -n^2 \langle \sigma_a v \rangle$$

Note the n^2 dependence

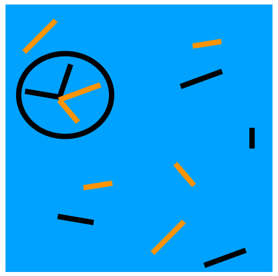
When the dark matter is in chemical equilibrium $n_{eq} = n$, which means there are an equal number of annihilation and creation events and

relativistic: $n_{eq} \sim T^3$ **(density is T^4)**

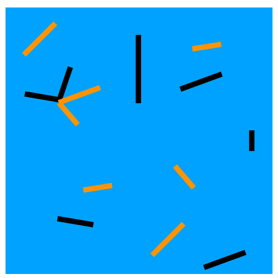
non relativistic: $n_{eq} \sim (m_x T)^{3/2} e^{-m_x/T}$ **(Boltzmann distribution)**

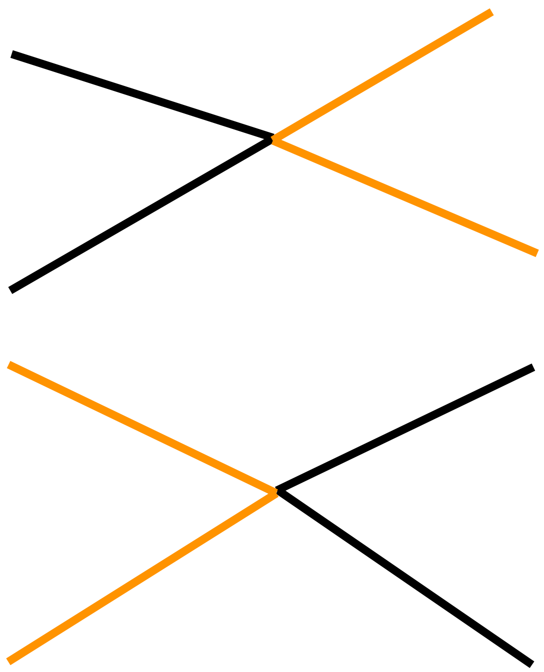
Put differently: dark matter production by other particles becomes extremely rare when the thermal bath temperature drops below the mass of the dark matter.

Is this process allowed by conservation of energy?



So if other (visible) particles are annihilating to dark matter, the dark matter particles will have an equilibrium number density that depends on whether DM is relativistic





$$\frac{dn}{dt} + 3Hn = \left(n_{eq}^2 - n^2\right)\langle\sigma_a v\rangle$$

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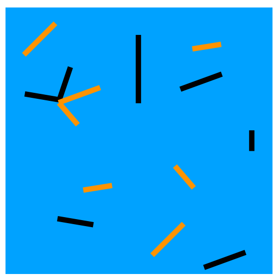
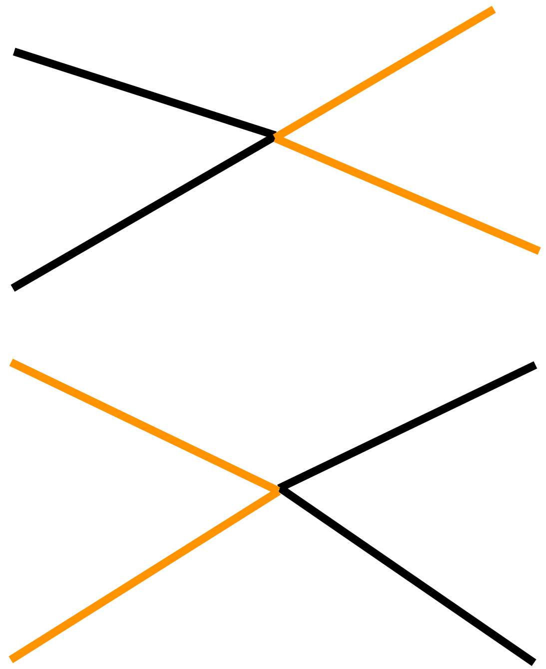


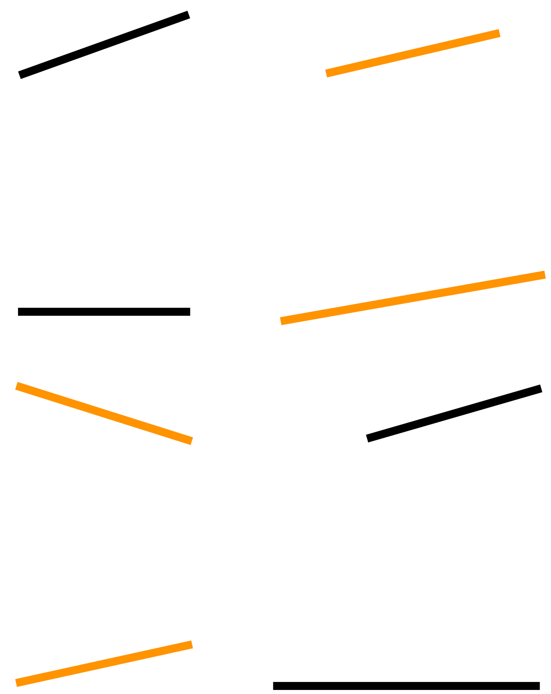


$$\frac{dn}{dt} + 3Hn = \left(n_{eq}^2 - n^2\right)\langle\sigma_a v\rangle$$

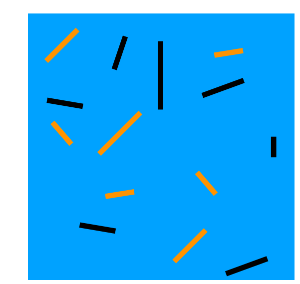
non relativistic: $n_{eq} \sim (m_x T)^{3/2} e^{-m_x/T}$ **(integrate Boltzmann distribution)**

relativistic: $n_{eq} \sim T^3$ **(density is** T^4**)**

Once the dark matter annihilation rate shuts off, $n^2 \langle \sigma_a v \rangle \sim 0$

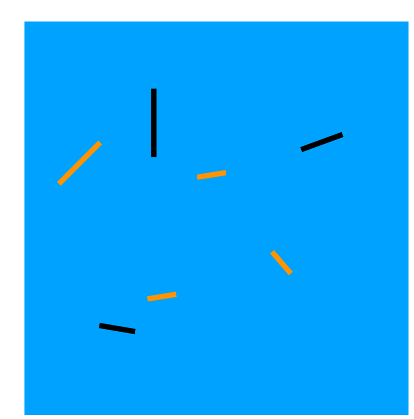


$$\frac{dn}{dt} + 3Hn = 0$$

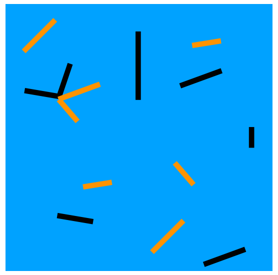


dark matter number density dilutes as the box expands,

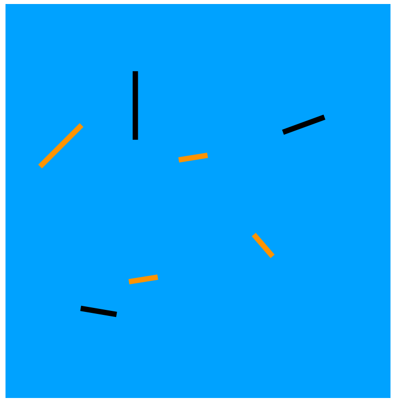
$$n \sim \frac{1}{a^3}$$



By inspection, the point at which annihilation/creation shuts off is



$$Hn \sim n^2 \langle \sigma_a v \rangle$$



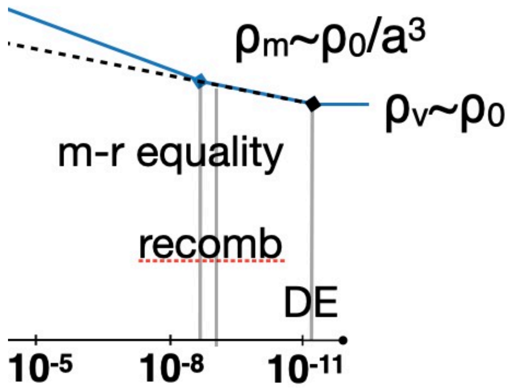
which is clear from

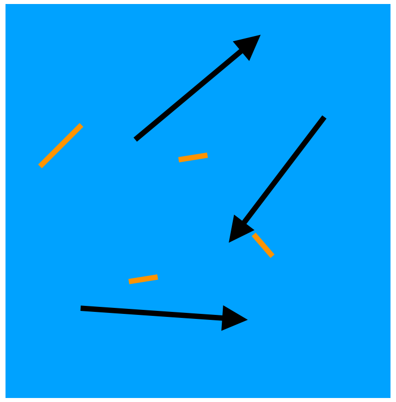
$$\frac{dn}{dt} + 3Hn = (n_{eq}^2 - n^2) \langle \sigma_a v \rangle$$

i.e. dark matter annihilation shuts off when the average time for annihilation is longer than a Hubble time.

$$1/t_H \equiv H \sim n \left\langle \sigma_a v \right\rangle$$

If dark matter is relativistic when it falls out of equilibrium, can it be our dark matter?





Hint: the requirement that it is relativistic when it decouples leads to a simple solution for what its mass must be, given that radiation-matter equality occurs at a temperature of an eV.

Another way to obtain the same answer is to follow this chain of logic:

1. The present baryon to photon ratio is around ~1:10^9

2. The present baryon to dark matter mass ratio is 1:5

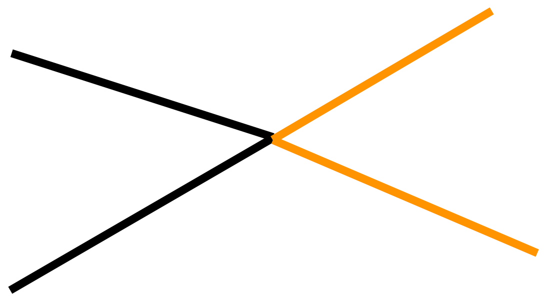
3. What is the current number density of photons versus dark matter if dark matter is relativistic at freeze out (extra hint: dark matter and SM photons will have the same number density if dark matter freezes out relativistically).

4. Given all the above, what is the implied dark matter mass?

5. How fast is that dark matter moving when the universe has density eV^4?

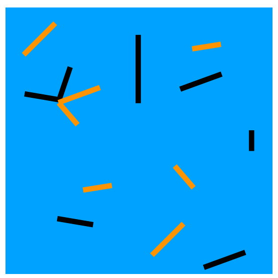
Calculation:

Now we can figure out the annihilation cross-section that gives the correct dark matter relic abundance for non-relativistic freeze-out



$\langle \sigma_a v \rangle$ **?**

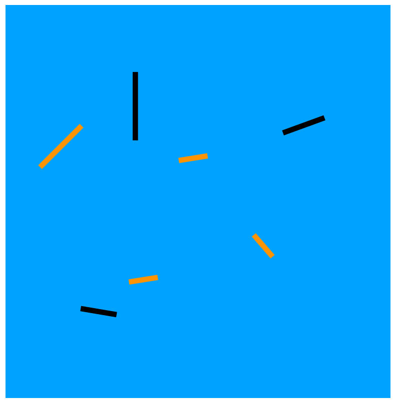
1. Begin with the freeze-out relation



$$Hn \sim n^2 \langle \sigma_a v \rangle$$

assume equilibrium freeze-out

$$H \sim n_{eq} \langle \sigma_a v \rangle$$



non-relativistic

$$n_{eq} \sim (m_x T)^{3/2} e^{-m_x/T}$$

^this is just $p^3=(mv)^3$, for $T=mv^2/2$

Calculation:

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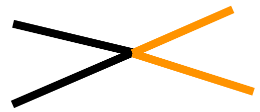

$\langle \sigma_a v \rangle$ **?**

2. Define the dimensionless freeze-out variable $x_f = m_x/T_f$, where the freeze-out temperature is T_f

3. Recall the formula relating the Hubble constant and density of the universe during radiation domination

$$m_{pl}^2 H^2 \sim \rho \sim T^4$$

4. Re-express in terms of x_f

$$H \sim \frac{m_x^2}{x_f^2 m_{pl}}$$

5. Combing this with (1.) we should find

$$e^{-x_f} \sim$$

Calculation:

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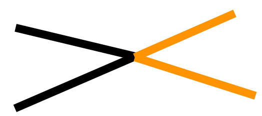

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5. Combing this with (1.) we should find

$$e^{-x_f} \sim \frac{x_f^{-1/2}}{m_x \left\langle \sigma_a v \right\rangle m_{pl}}$$

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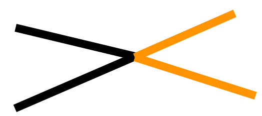

$\langle \sigma_a v \rangle$ **?**

6. Because x_f is small and an exponential scales faster than a power law we can approximate,

$$x_f \sim log(m_x m_{pl} \langle \sigma_a v \rangle)$$

7. Now we'll find another expression for x_f. Using that before the temperature of m-r equality, DM density ~T^-3,

$$\rho_{dm} = m_x n_{eq} \sim T_f^3 T_r$$

$$\langle \sigma_a v \rangle \sim \frac{m_x^2}{x_f^2 m_{pl} n_{eq}} \sim \frac{m_x^3}{x_f^2 m_{pl} \rho_{DM}} \sim \frac{x_f}{T_r m_{pl}}$$

we see that the freeze-out temperature depends only logarithmically on the dark matter mass

Calculation:

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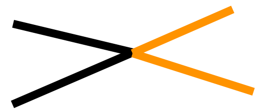


$\langle \sigma_a v \rangle$ **?**

8. Now we solve for the annihilation cross-section in terms of x

$$\langle \sigma_a v \rangle \sim 10^{-10} \times x_f \times \mathrm{GeV}^{-2}$$

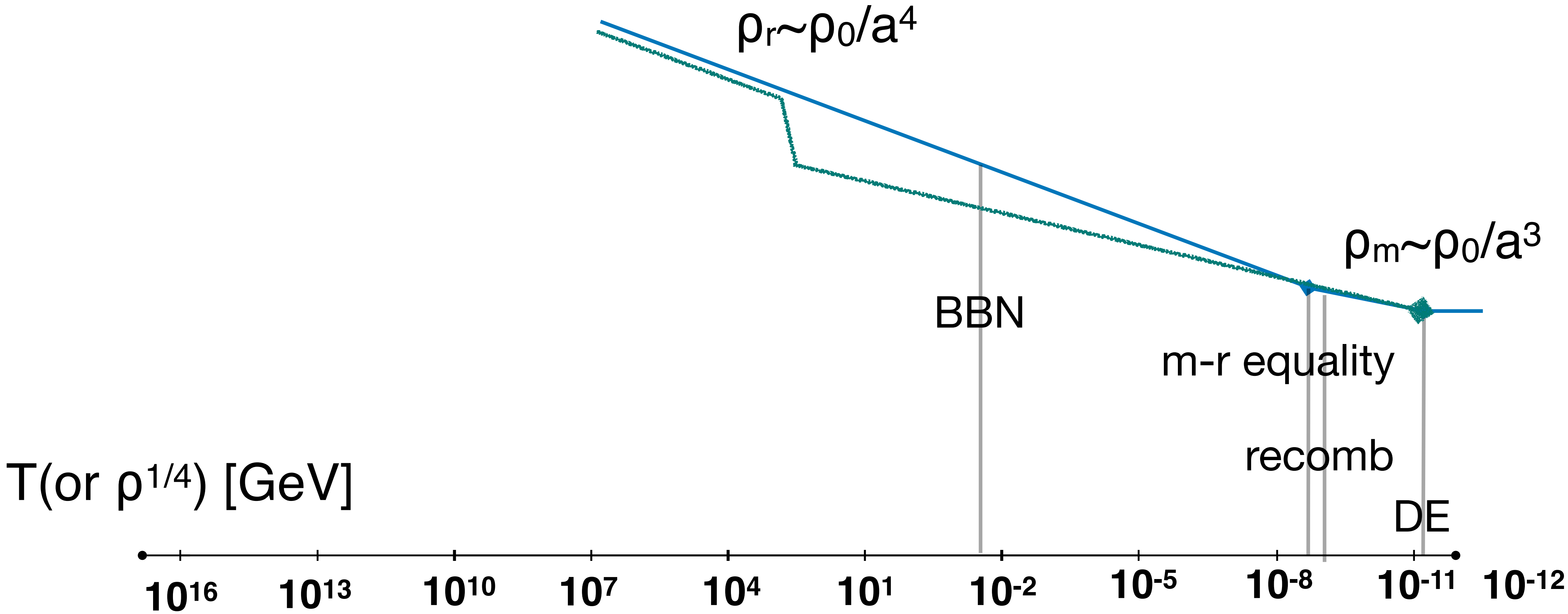
9. Use 6. and 8. to determine, x_f. Use x_f ~10 as a test value if necessary. Substitute into the logarithmic expression to find x_f.

$$x_f \sim 25 - 30 \text{ for } m_x \sim 10 - 10^3 \text{ GeV}$$

10. Now find the annihilation cross-section using x_f

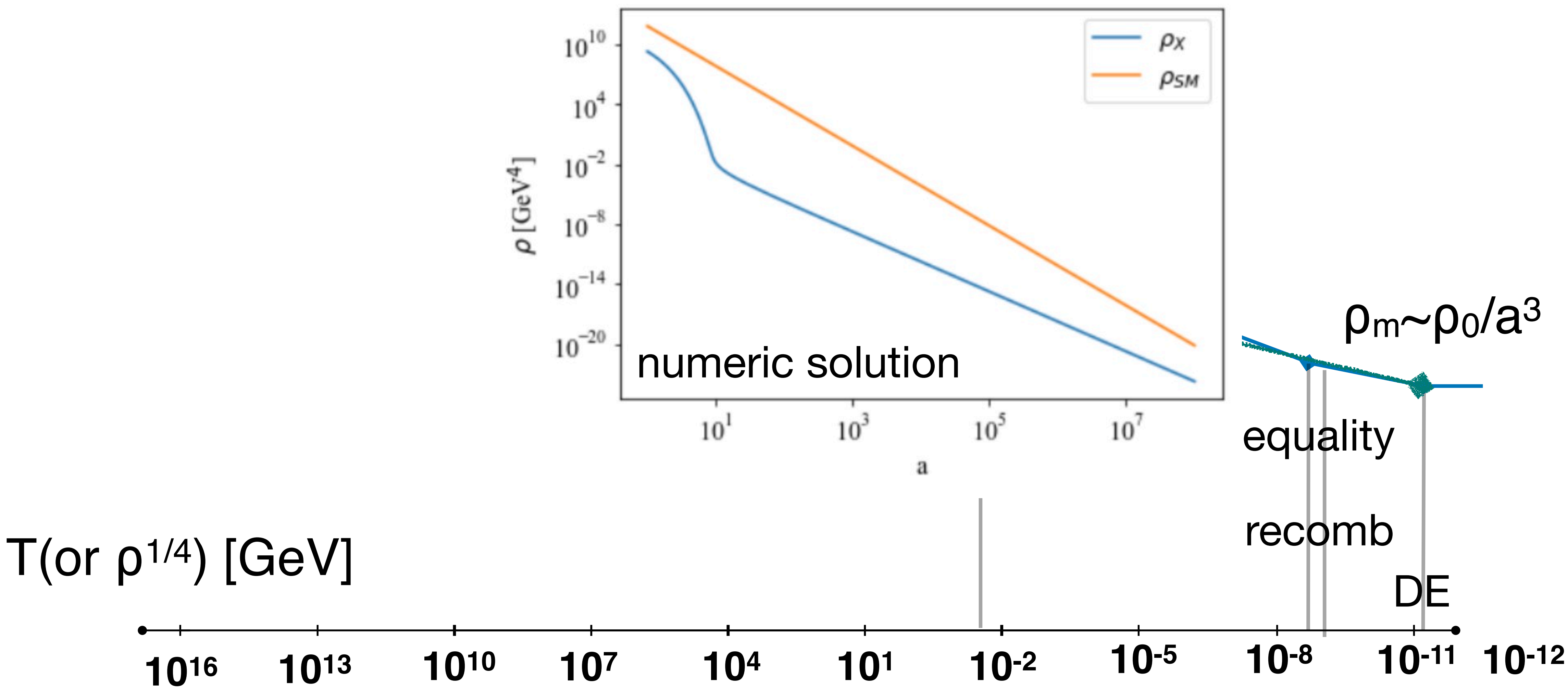
$$\langle \sigma_a v \rangle \sim 3 \times 10^{-26} \ \frac{\mathrm{cm}^3}{s} \sim 10^{-36} \ \mathrm{cm}^2$$

Freeze-out



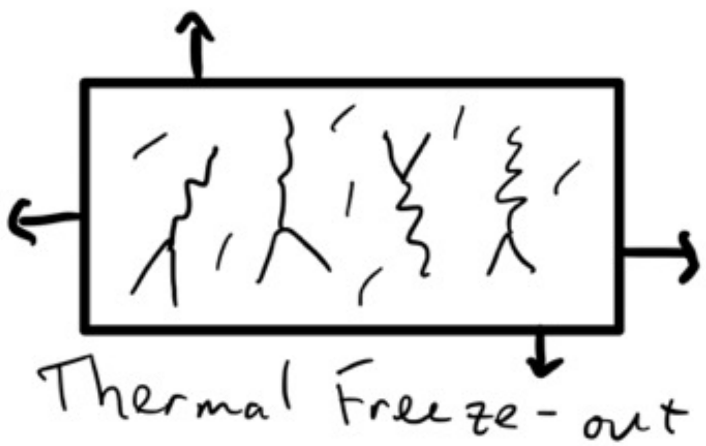
ρr~ρ0/a4
ρm~ρ0/a3
BBN
m-r equality
recomb
DE
T(or ρ1/4) [GeV]
1016
1013
1010
107
104
101
10-2
10-5
10-8
10-11
10-12

Freeze-out



ρ_X
ρ_{SM}
ρ [GeV^4]
10^{10}
10^{4}
10^{-2}
10^{-8}
10^{-14}
10^{-20}
10^{1}
10^{3}
10^{5}
10^{7}
a
numeric solution
$\rho_m \sim \rho_0/a^3$
equality
recomb
DE
T(or $\rho^{1/4}$) [GeV]
10^{16}
10^{13}
10^{10}
10^{7}
10^{4}
10^{1}
10^{-2}
10^{-5}
10^{-8}
10^{-11}
10^{-12}

Weakly interacting dark matter "miracle"



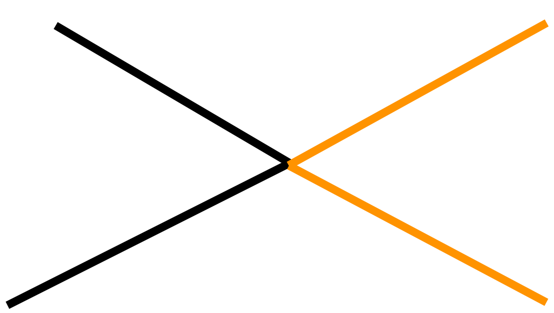
As the universe cools, dark matter falls out of thermal equilibrium, some portion annihilates to SM particles

dm abundance: $\langle \sigma_a v \rangle \sim \frac{x_f}{T_r m_{pl}}$

The final relic abundance depends on the annihilation cross-section, but only logarithmically on m_x

$$x_f \sim log(m_x m_{pl} \langle \sigma_a v \rangle)$$

$$\langle \sigma_a v \rangle \sim 3 \times 10^{-26} \ \frac{\mathrm{cm}^3}{s} \sim 10^{-36} \ \mathrm{cm}^2$$

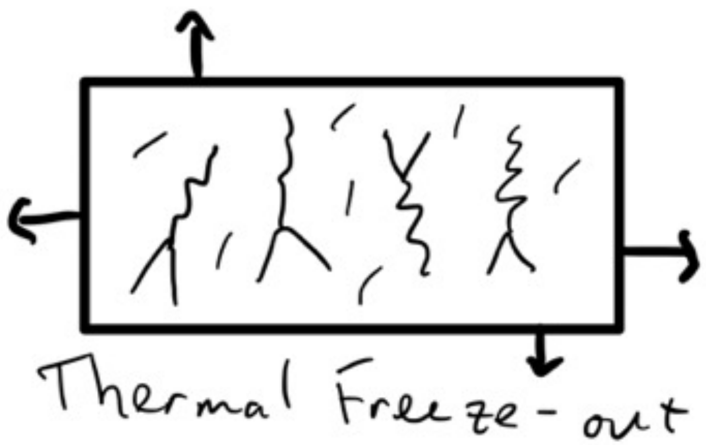


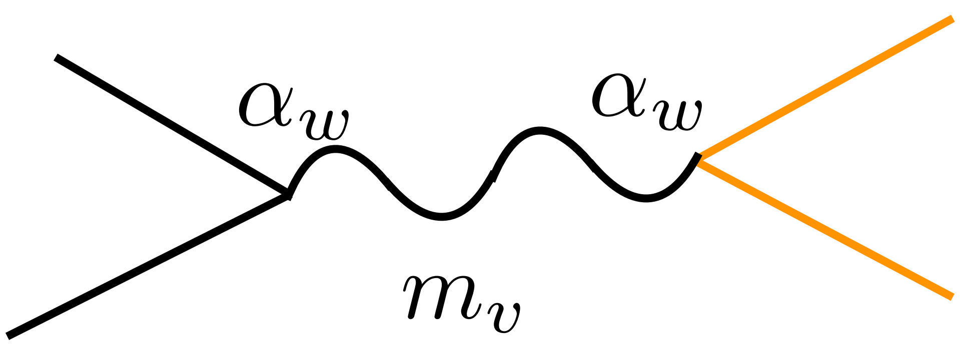
Taking the annihilation cross-section to be

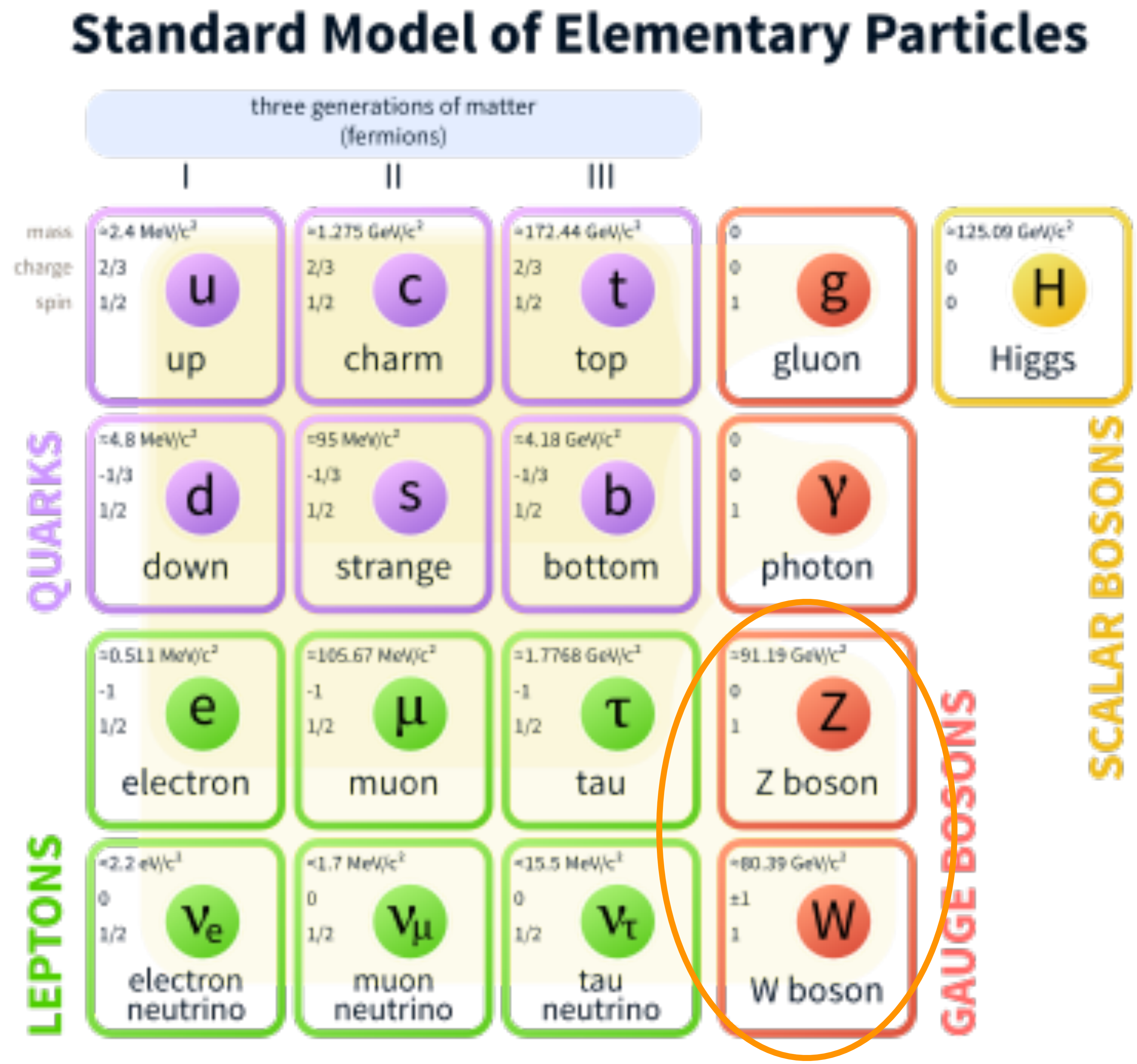
$$\sigma_a \sim \frac{\alpha^2}{m^2}$$

what is m for alpha=1 and alpha=0.03?
(feel free to take v~1 in the above)

WIMP Miracle







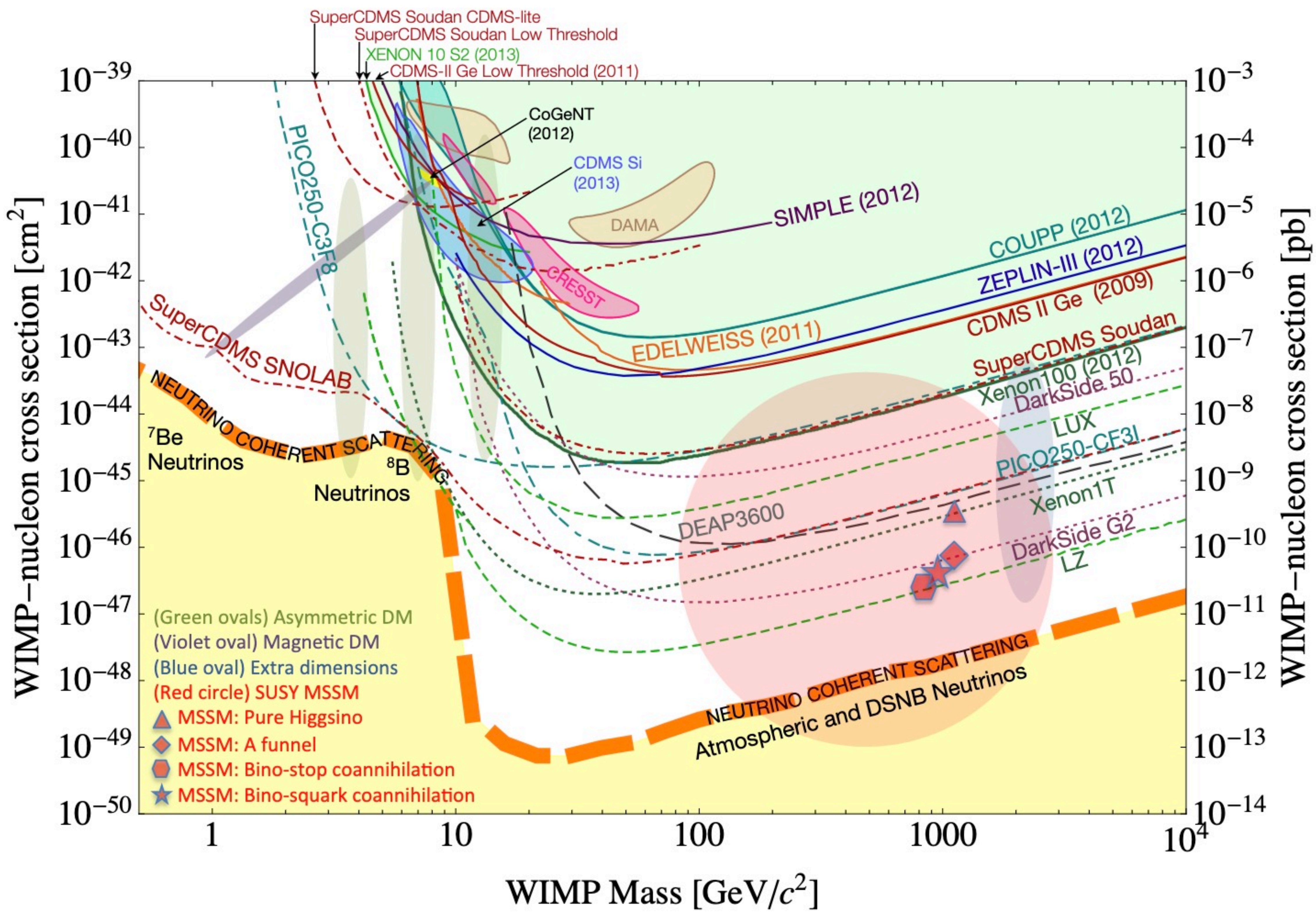
$$\Omega_x h^2 \sim 0.1 \left(\frac{m_{\rm v}}{100\ {\rm GeV}}\right)^2 \left(\frac{0.03}{\alpha_w}\right)^2$$

The thermal relic annihilation cross-section roughly matches the couplings and mass of the weak force, "wimp miracle"

The 100 TeV Mass Unitarity "Limit" or Why do the plots stop at $m_X \sim 10$ TeV?

'87

1. A



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2. R

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free is

The 100 TeV Mass Unitarity "Limit" or Why do the plots stop at $m_X \sim 10$ TeV?

Griest, Kamionkowski, '87

1. Assume freeze-out abundance set with annihilation

$$\sigma_a \sim \text{picobarn} = 10^{-36}\ \text{cm}^2$$ {wimp miracle

2. Require the annihilation cross-section not exceed a perturbative bound

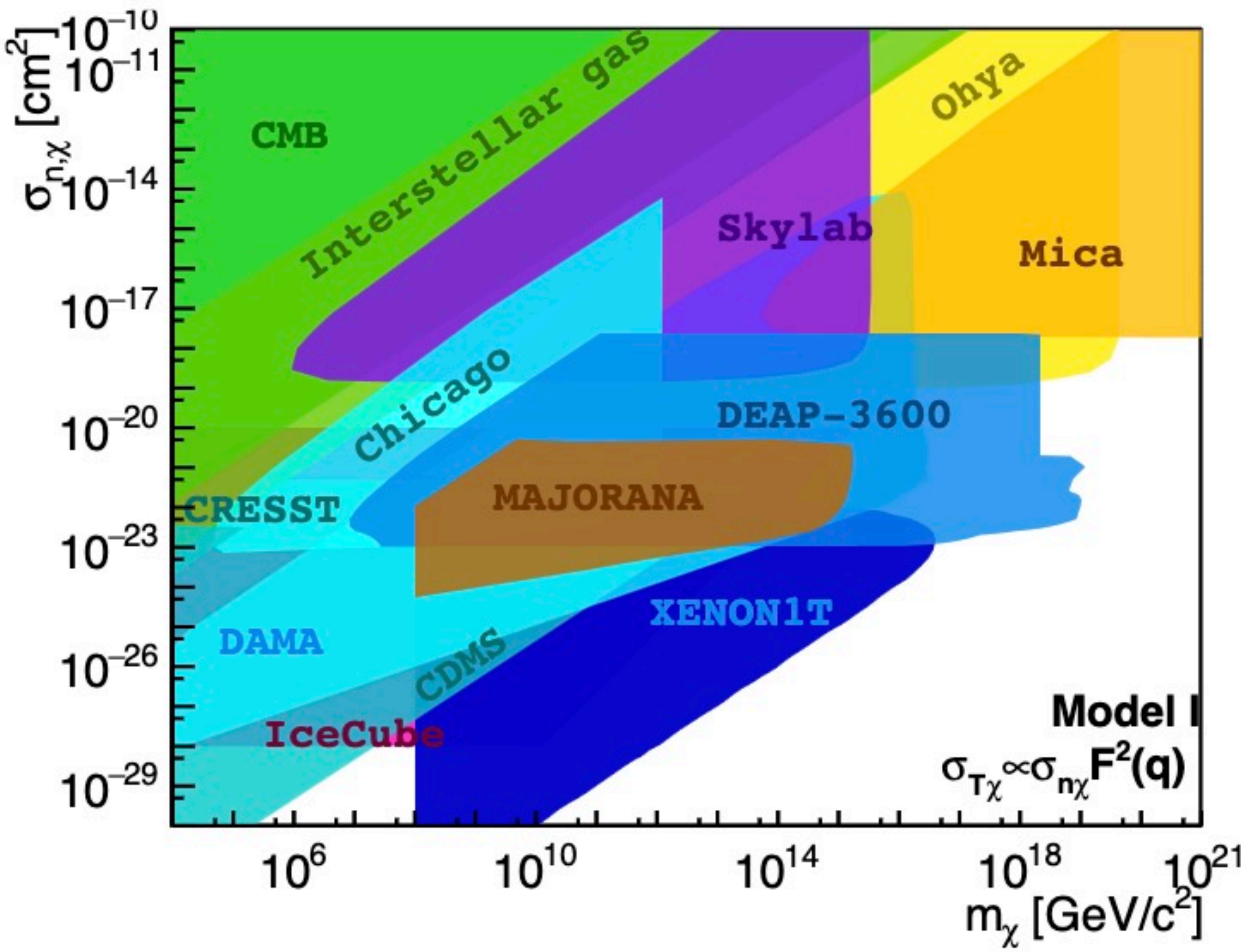
$$\sigma_a \lesssim 4\pi/m_x^2$$

3. Then because this cross-section is a picobarn for thermal freeze-out, the suggestion for frozen out dark matter mass is

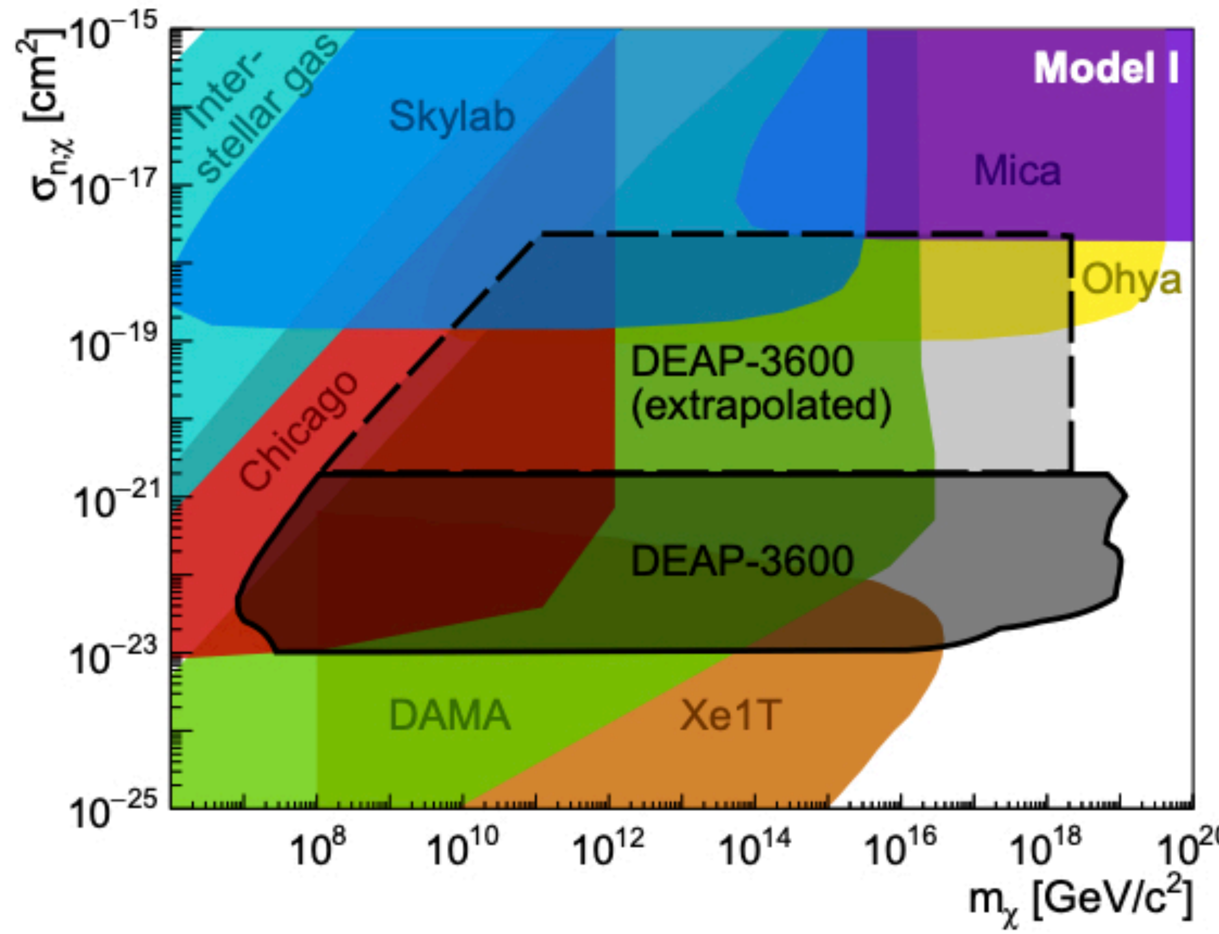
$$m_x \lesssim 10^5\ \text{GeV}$$

The 100 TeV Mass Unitarity "Limit" or Why ~~do~~ did the plots stop at $m_\chi \sim 10$ TeV?

Griest, Kamionkowski, '87



Snowmass 2021



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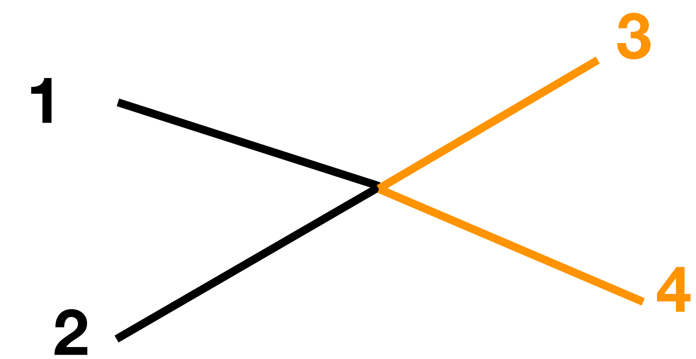
Where did the baryons come from?



$$\mathbf{L}[f] = \mathbf{C}[f]$$

$$\mathbf{C}[f(p_1)] = -\frac{1}{E_1}\int\prod_{i=2}^{4} dP_i\,(2\pi)^4\,\delta^{(4)}(p_1+p_2-p_3-p_4)\Big(f_1 f_2(1\pm f_3)(1\pm f_4)|\mathscr{M}(12\to 34)|^2$$
$$-f_3 f_4(1\pm f_1)(1\pm f_2)|\mathscr{M}(34\to 12)|^2\Big)$$

Sakharov Conditions:

1. Baryon number violation

   Minimum requirement

2. Charge and Charge-Parity violating process

   Otherwise time reversal symmetry washes out asymmetry

3. Out of equilibrium process

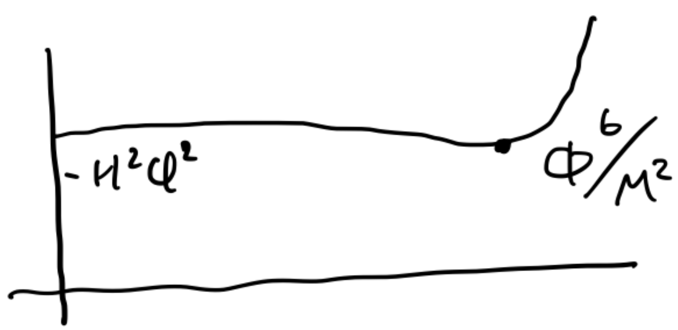
   Else reverse process washes out asymmetry

AD Baryogenesis

Afleck, Dine ’85
Linde ’85
Dine, Randall, Thomas ‘95

1. Baryo-charged scalar gets vev during inflation

$$V_{AD} = m_\phi^2 |\phi|^2 - H^2 |\phi|^2 + \frac{\phi^6}{M^2} + \cancel{CP}$$



2. Baryo-charged scalar decays (cp-violating)

after inflation

$H \lesssim m_\phi$

$$(aH + b\, m_\phi)\, \phi^4/M + h.c.$$

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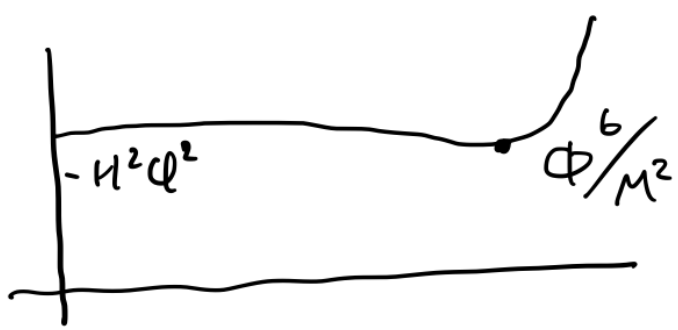

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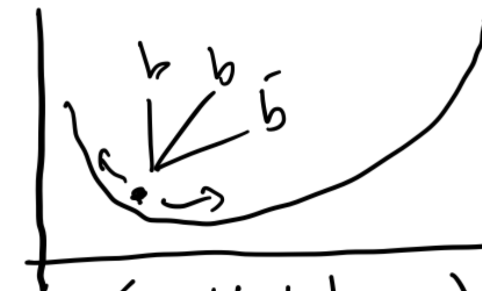


$$(aH + b m_\phi)\,\phi^4/M + h.c.$$

3. Oops, too many baryons, need → dilution

Main point **$\eta_b \sim 1$** for a simple baryo-charged scalar

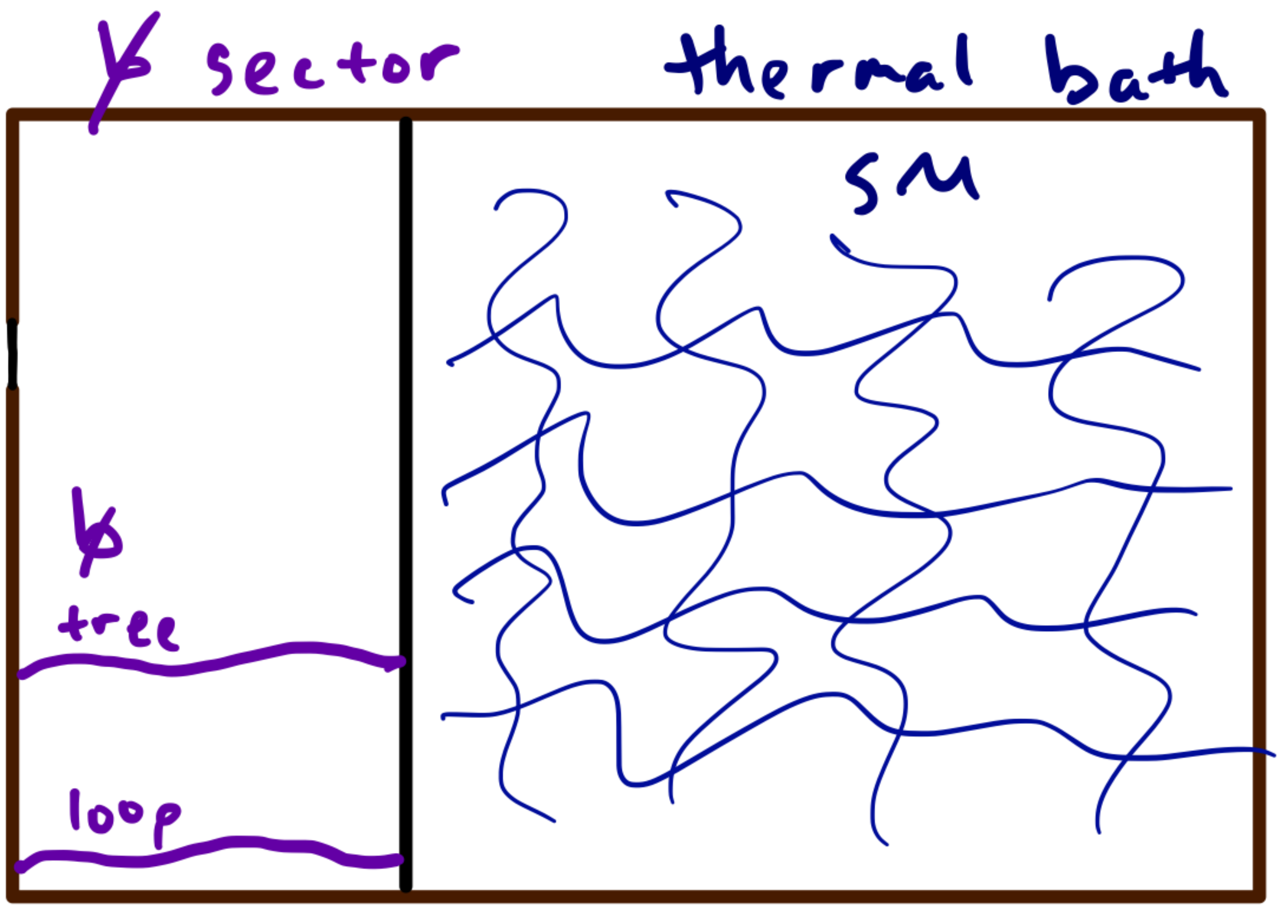
$\eta_b \sim 10^{-10}$ observed, need dilution.

Why too many baryons?

If: $\mathcal{O}(1)$ CP violating decays

And: $\rho_\phi \sim \rho_u$

$\not B$ sector



$$\eta_B \simeq \frac{n_b}{\rho_u^{3/4}}\left(\zeta\right)$$

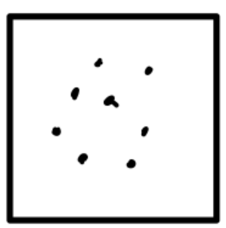
$n_b \sim [10^{-5} - 1]$ for any $\not B$ sectors with $\mathcal{O}(1)$ couplings

High Scale Baryon Asymmetry Cosmology Often Has Dilution

1. Stat. field

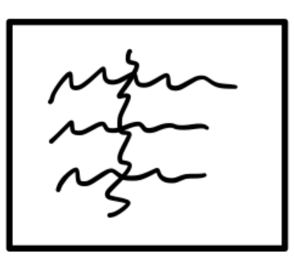
2. matt. field

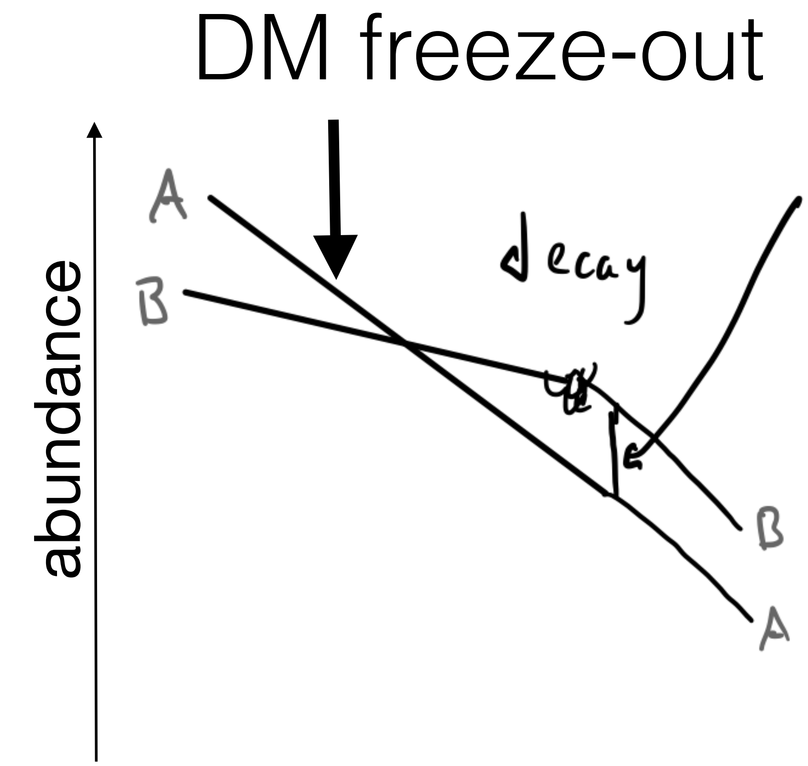
or



$\Gamma < H$

3. rad. field





$$\zeta \equiv \frac{\rho_A^{3/4}}{\rho_B^{3/4}}$$

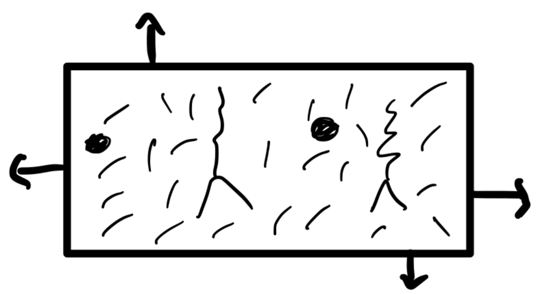
$$\Omega h^2 \to \Omega h^2 \zeta$$

<u>dilution</u>

- **B** : slow decaying inflatons, moduli, GUT mass states, AD fields...

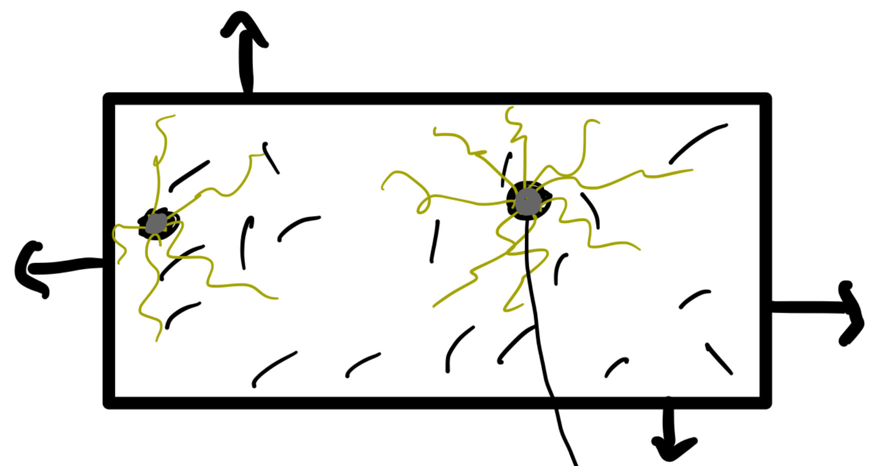
- **A** : Typically SM w/DM, baryons

Diluted Dark Matter



Overabundant freeze-out

Then dilution from decay →

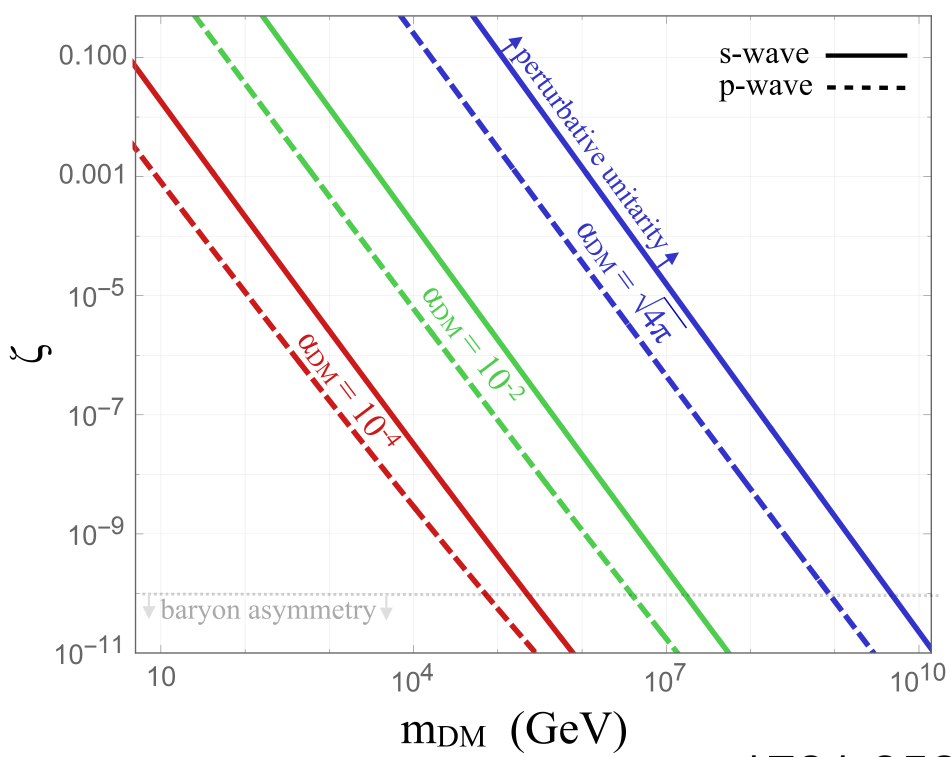


Motivation

$$\Omega_x h^2 \sim 0.1 \left(\frac{m_V}{PeV}\right)^2 \left(\frac{0.03}{\alpha_D}\right)^2 \left(\frac{\zeta}{10^{-8}}\right)$$

- -Matter dominated epoch
- -Decay of asymmetry field (Affleck-Dine)
- -Decay of inflaton
- -Decay of modulus / gravitino
- -Field associated with ~PeV dark sector

$$\boxed{\zeta \equiv \frac{s_{ini}}{s_{fin}} = n_x \text{ dilution}}$$



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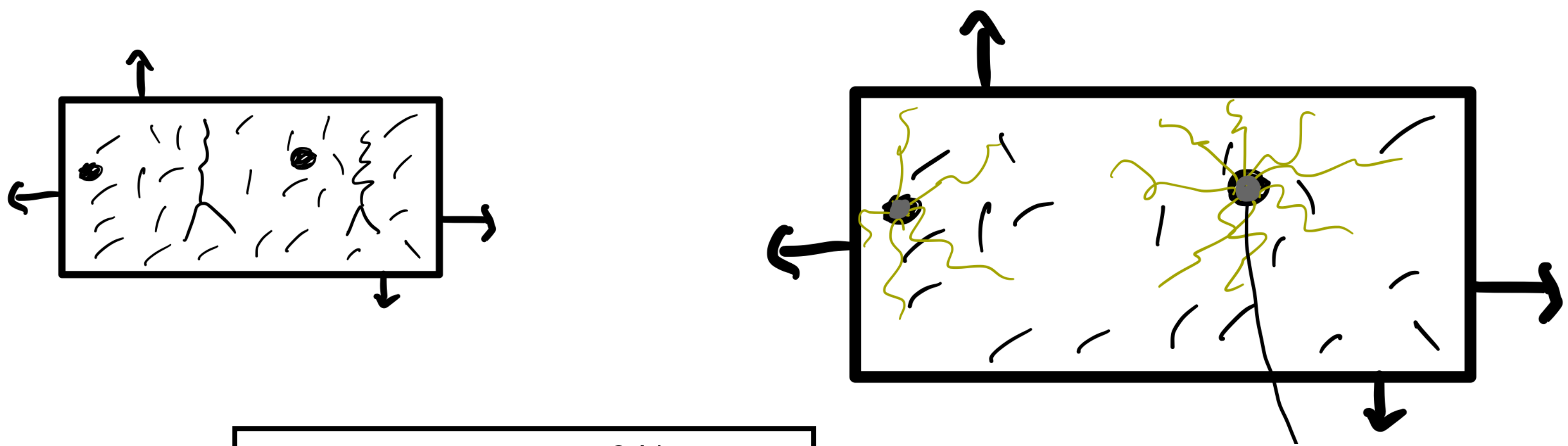
see also e.g.
Affleck Dine '85
Allahverdi Dutta Sinha '11
Kane Shao Watson '11
Davoudiasl Hooper McDermott '15
Berlin Hooper Krnjaic '16

Cosmological dilution - super-WIMP DM



$$\boxed{\zeta \equiv \frac{s_{ini}}{s_{fin}} \sim \frac{\rho_{decay}^{3/4}}{\rho_{rad}^{3/4}}}$$

Exercise: revisit the relativistic freeze-out dark matter calculation with dilution

Assume dark matter falls out of equilibrium when the radiation bath T~m_X~TeV, and so maintains the same number density as fields in the radiation bath

If a heavy state decays at T~ 10 GeV, what does ρ_{decay} need to be in relation to ρ_{rad} in order produce the observed dark matter?

Asymmetric Dark Matter

-Asymmetric dark matter (like ordinary matter) composed of particles and not antiparticles under some symmetry.

$$\phi \, \phi^* \quad \text{or} \quad \psi \, \bar{\psi}$$

-For asymmetric DM and for baryons/leptons in the Standard Model, freeze out annihilation cannot annihilate the residual "asymmetric" part — the asymmetric part is left over after freeze out finishes.

$$Y_{asymm,d} \equiv \frac{n_{\bar{x}}}{s} = \eta_D$$

asymmetry is fixed
to provide correct DM abundance

- If we assume a cold freeze-out abundance for the dark matter, the requirements that we annihilate away the "symmetric component," since we set the dark matter abundance using the dark sector asymmetry (just like the SM baryon abundance)

$$\langle \sigma_a v \rangle \gg 3 \times 10^{-26} \ \frac{\text{cm}^3}{s} \sim 10^{-36} \ \text{cm}^2$$

Asymmetric Dark Matter

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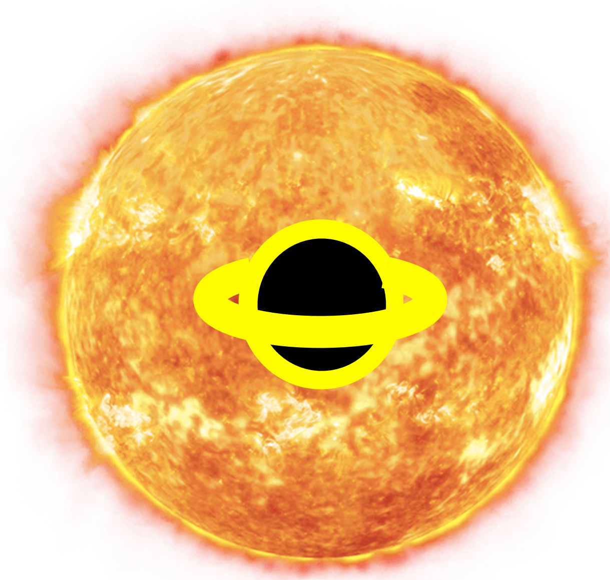
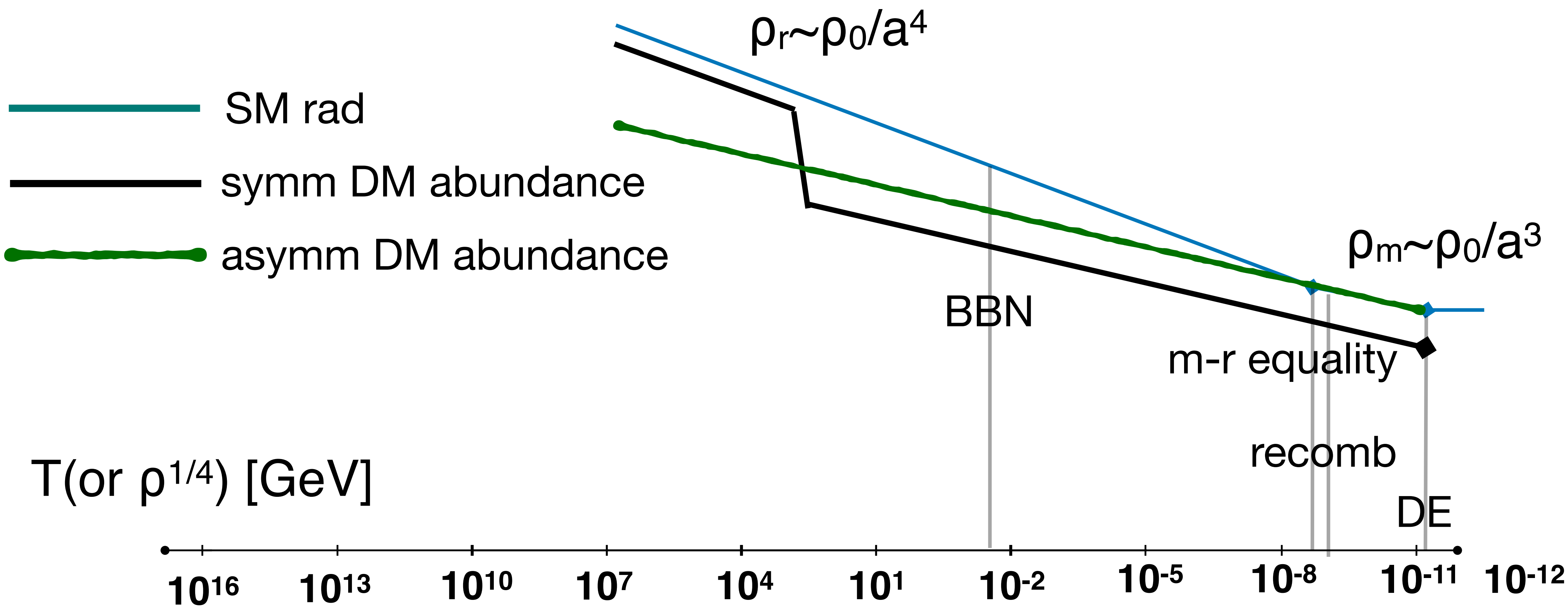
-For asymmetric DM and for baryons/leptons in the Standard Model, freeze out annihilation cannot annihilate the residual “asymmetric” part — the asymmetric part is left over after freeze out finishes.

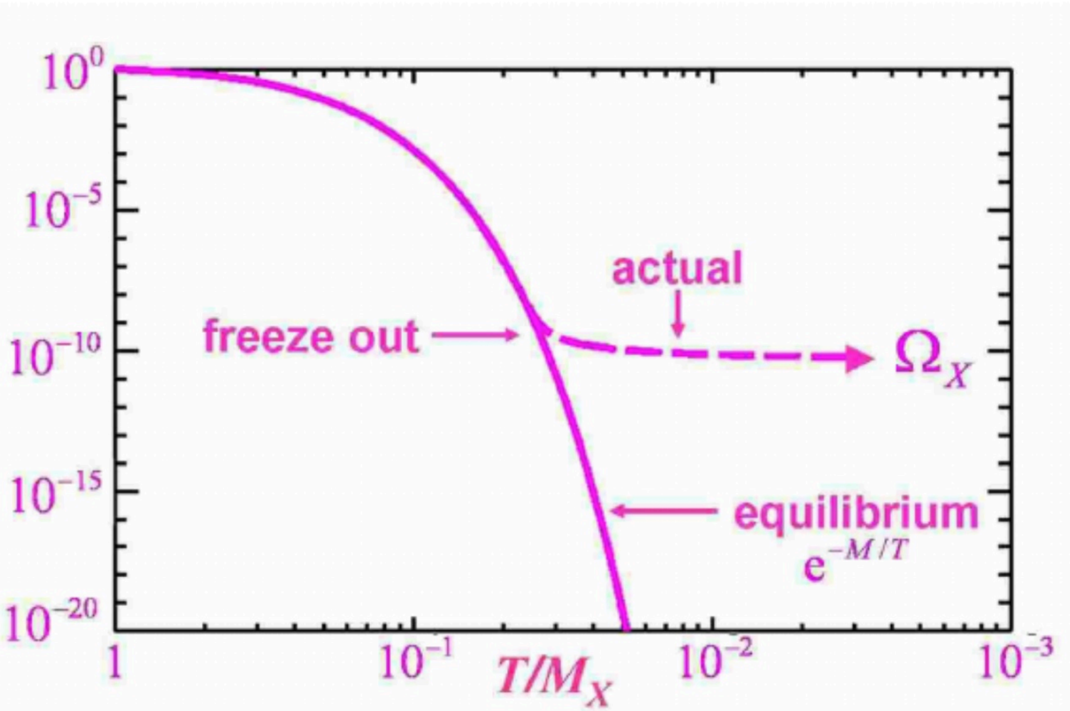
-Dark matter asymmetry allows collection and collapse in stars/planets without annihilating to lighter particles.



Asymmetric Dark Matter



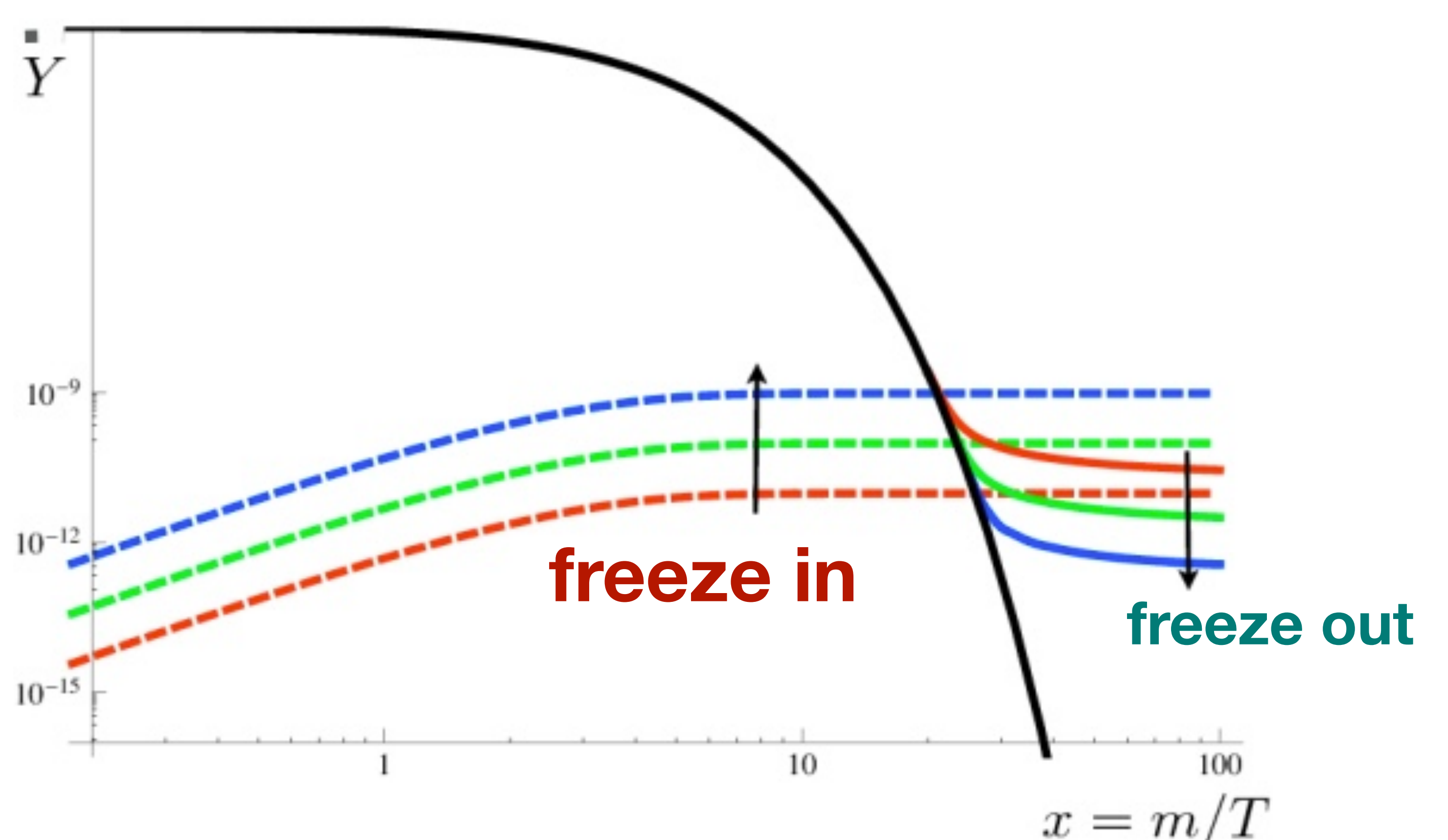
Freeze Out



Dark Matter is in thermal equilibrium with SM, plenty of interactions between the two sectors.

Freeze In

Dark matter begins "out of equilibrium" and slowly grows in abundance, in this case the DM is very weakly coupled to the Standard Model, producing DM in the thermal bath is a rare process.



Freeze In Calculations

For freeze-in the Boltzmann equations are simpler - drop piece that tracks DM annihilation to SM, since this should be negligible

$$\boxed{\dot{n} + 3Hn = \cancel{-\int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3} f_1 f_2 (\sigma v_{\rm Moll})_{12\to 34}} + \int \frac{d^3p_3}{(2\pi)^3}\frac{d^3p_4}{(2\pi)^3} f_3^{\rm eq} f_4^{\rm eq} (\sigma v_{\rm Moll})_{34\to 12}}$$

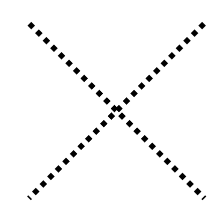
A very crude estimate of the number density of freeze-in dark matter can be obtained by comparing the velocity averaged production rate to Hubble for freeze-in dark matter.

$$Y_x \equiv \frac{n_x}{s} \sim \frac{<\sigma v>}{sH} \sim \frac{\sigma}{H} T^3$$

The rate for freeze-in production will depend on the dark matter's coupling to the SM thermal bath.

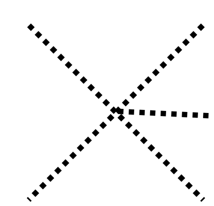
For example for a quartic scalar coupling, $\sigma \sim \dfrac{\lambda^2}{T^2}$

$$Y_x \sim \frac{\lambda^2 m_{pl}}{T}$$



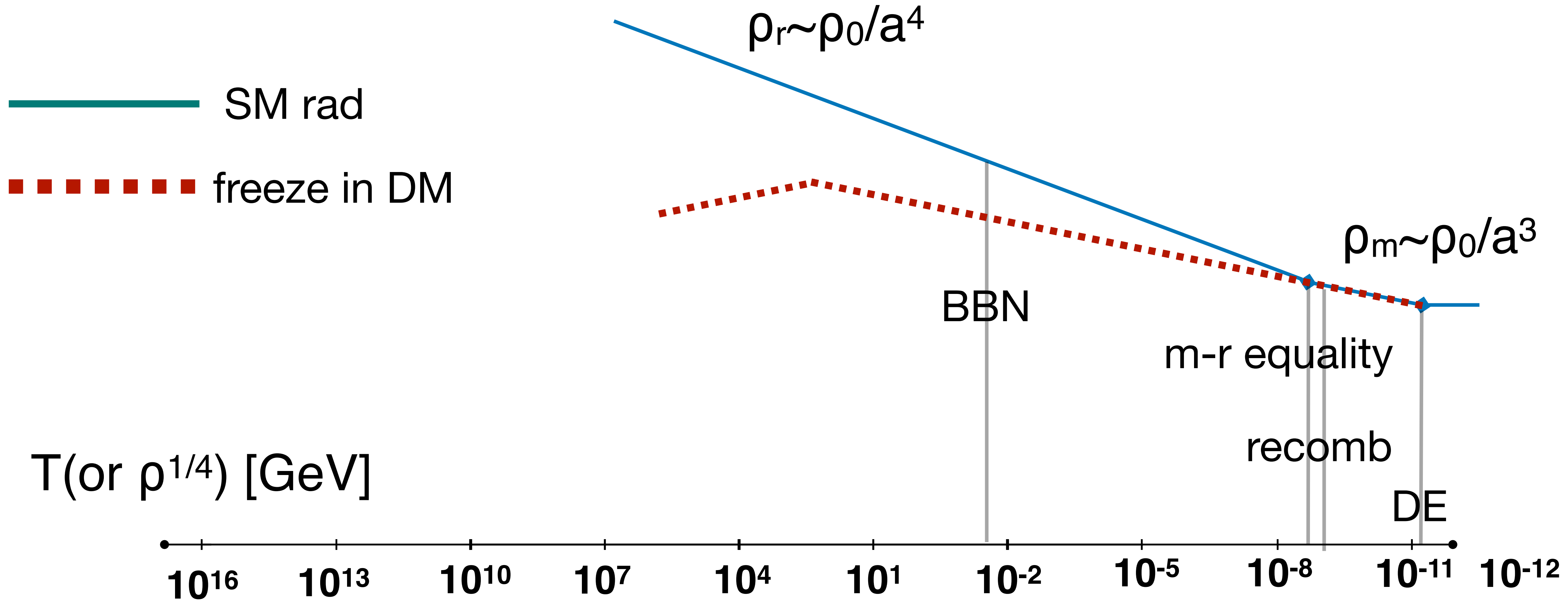
For an effective operator coupling $\sigma \sim \dfrac{1}{\Lambda^2}$

$$Y_x \sim \frac{T m_{pl}}{\Lambda^2}$$



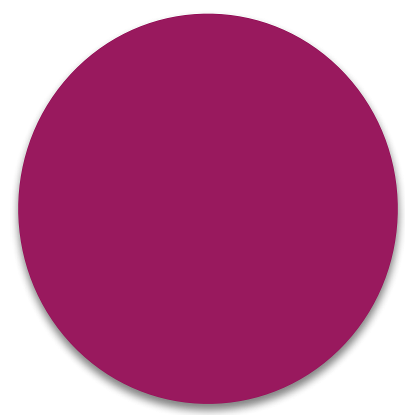
Exercise: for the scalar quartic model above, solve for λ that gives the correct dark matter relic abundance.

Freeze In


SM rad
freeze in DM
ρr~ρ0/a4
ρm~ρ0/a3
BBN
m-r equality
recomb
DE
T(or ρ1/4) [GeV]
1016
1013
1010
107
104
101
10-2
10-5
10-8
10-11
10-12

Now let's consider an overdensity in the early thermal bath of the universe

How might it collapse to form a black hole?



Free fall time

$$t_{ff} \sim \frac{1}{\sqrt{G\rho}}$$

combine these
for

Jean's criterion

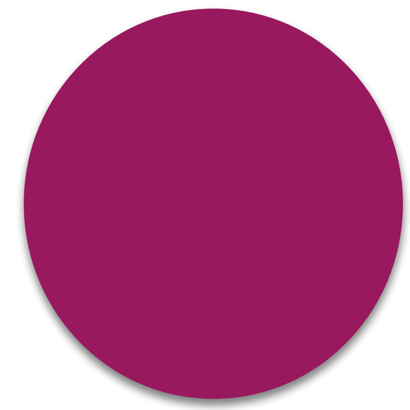
$$M \sim \frac{c_s^2 R}{G}$$

Sound crossing time

$$t_{sc} \sim \frac{R}{c_s}$$

Jean's criterion in the early universe

The mass contained in a Hubble horizon determines
the mass that can collapse to form a black hole for R = 1/H.



$$M \sim \frac{c_s^2 R}{G} \sim \frac{c_s^2}{GH}$$

Using what we know about cosmology, what is the mass of the primordial black hole, as a function of the thermal bath temperature? Take $c_s \sim 1$. Hint: use the relation $3H^2 m_{pl}^2 = T^4$.

Jean's criterion in the early universe

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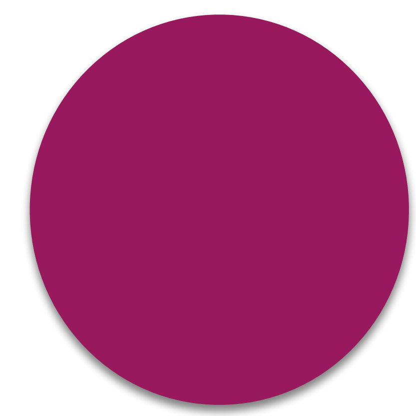

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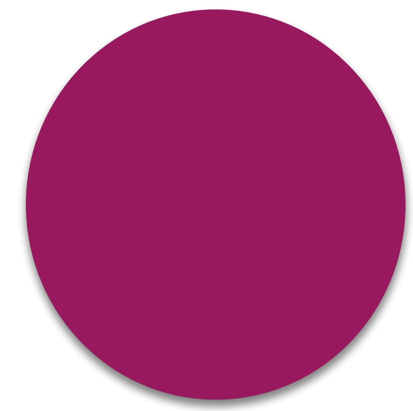
Using what we know about cosmology, what is the mass of the primordial black hole, as a function of the thermal bath temperature? Take c_s ~ 1. Hint: use the relation $3H^2 m_{pl}^2 = T^4$.

$$M_{pbh} \sim 10^{52}\ \mathrm{GeV}\ \left(\frac{100\ \mathrm{GeV}}{T}\right)^2$$

Hawking Radiation

First, let's determine the Schwarzschild Radius as the radius of no escape, assuming an escape velocity of c. The answer should be in terms of fundamental constants and the BH mass.

$$\frac{1}{2}mv^2 = \frac{2GMm}{R} \quad \Big| \quad v = \sqrt{\frac{2GM}{R}} = 1 \quad \Big| \quad R_{schw} = 2GM$$



Next, let's figure out, if a black hole radiated, what would be its typical wavelength/temperature? Hint: What is the only quantity that defines a simple black hole?

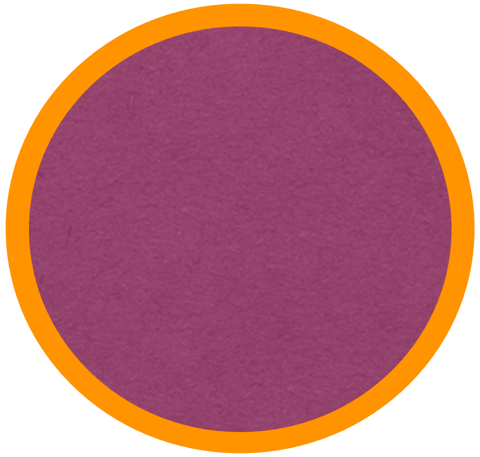
$$T \sim \frac{1}{\lambda} \sim \frac{1}{R} = \frac{1}{2GM}$$

Plug this information into the blackbody power formula:

$$P = \frac{2\pi^5}{15} A T^4$$

Now that we have our black hole radiating, let's figure out how quickly it would evaporate, as a function of its mass.

$$P = \frac{dM}{dt} = \frac{1}{15360\pi G^2 M^2}$$



How massive would it have to be to not disappear in a billion years?

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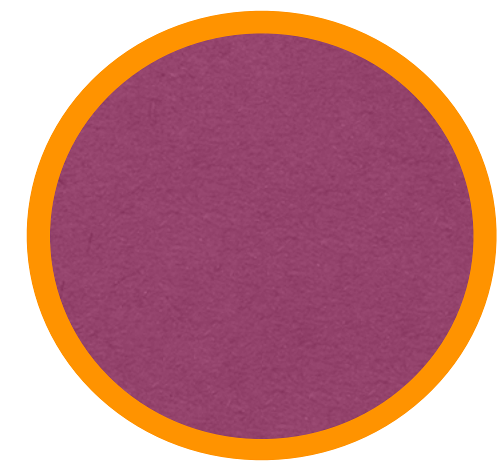
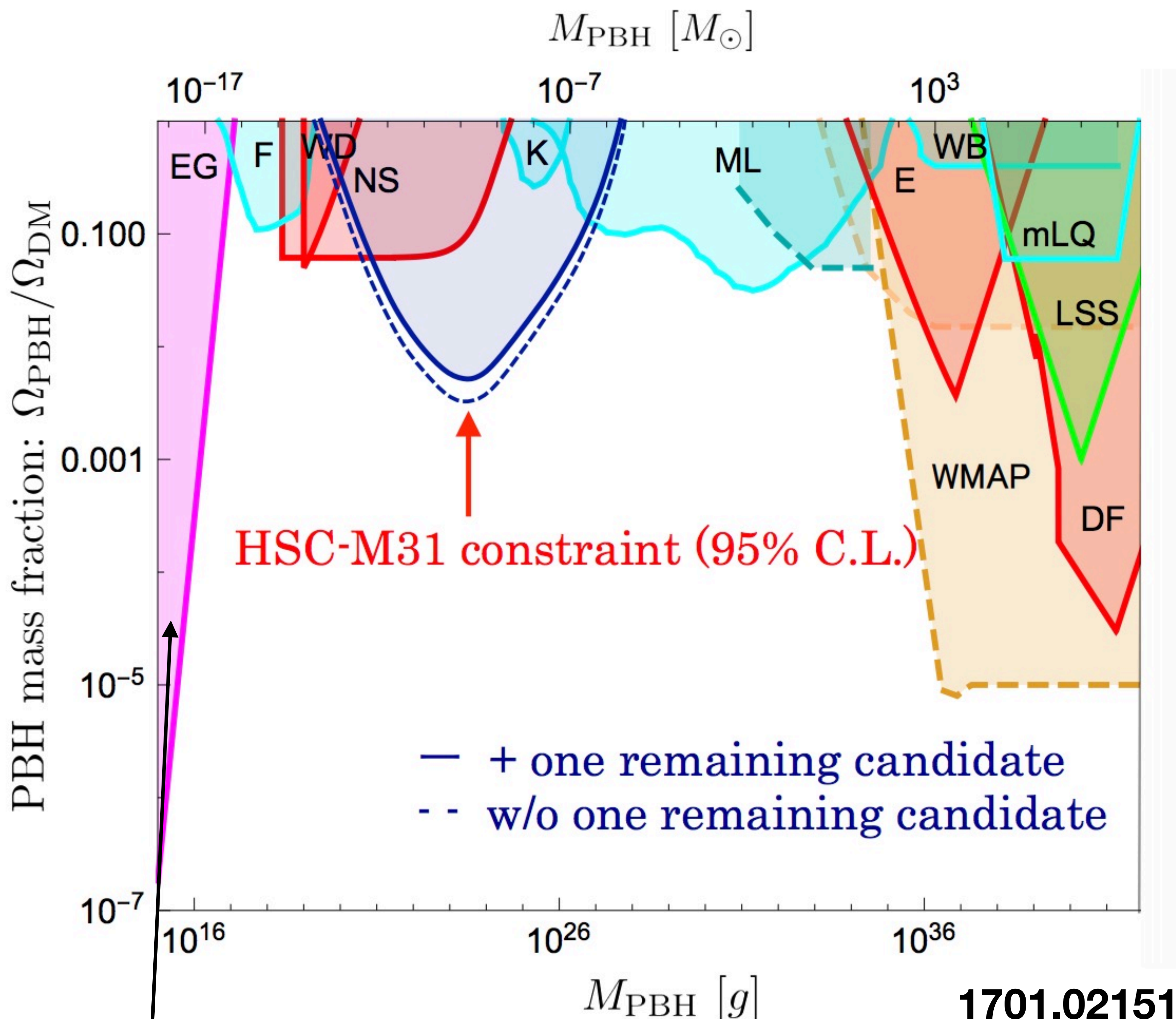


How massive would it have to be to not disappear
in a billion years?

$$t_{evap} \sim 10^9 \text{ years} \left(\frac{M}{10^{38} \text{ GeV}} \right)^3 \sim 10^9 \text{ years} \left(\frac{M}{10^{14} \text{ grams}} \right)^3$$

Constraints on primordial black holes



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Notice the evaporation bound!

Capstone exercise II: make your own dark matter

Choose a kind of dark matter production scenario, or a combination of scenarios.
(If you feel you have mastered dark matter production, try a diluted model.)

Specify a certain mass and figure out the couplings for your dark matter model.

- e.g. for freeze-in, find the weak coupling necessary for the right DM abundance
- for asymmetric dark matter, specify the asymmetry and minimum

Is your dark matter wavelike or particle-like? Are there any ways you can think of for detecting it?