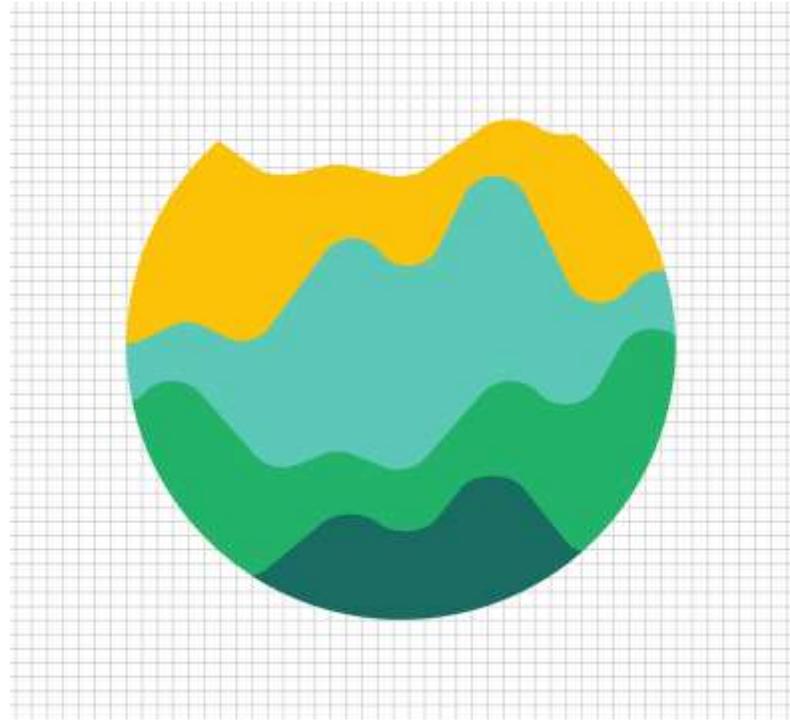


# Long Baseline Neutrino Oscillation Lecture 1



TRISEP 2024

Tuesday July 16, 2024

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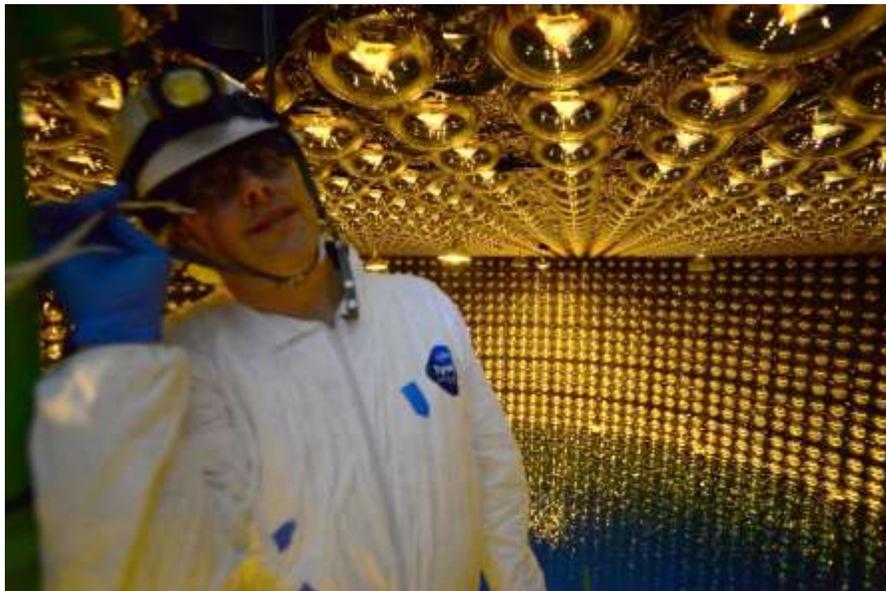
THE UNIVERSITY OF WINNIPEG

# A brief Bio

- Engineering Physics (computing+electrical) UBC 1997
- MSc and BEd 1998-2001
- PhD -- TWIST muon decay Michel parameters 2006
- Postdoc -- SNO NCD solar flux joint fit NCD + Cherenkov 2007-2009
- Postdoc on T2K – first TPC test at TRIUMF for the near detector
- Faculty at University of Winnipeg
  - 2011-2020 – Ultracold neutrons + T2K electron neutrino xsec
  - 2021-current – research towards CP violation in Hyper-K



Me at Super-Kamiokande



Me at T2K ND280 near detector



# Lecture 1 Outline



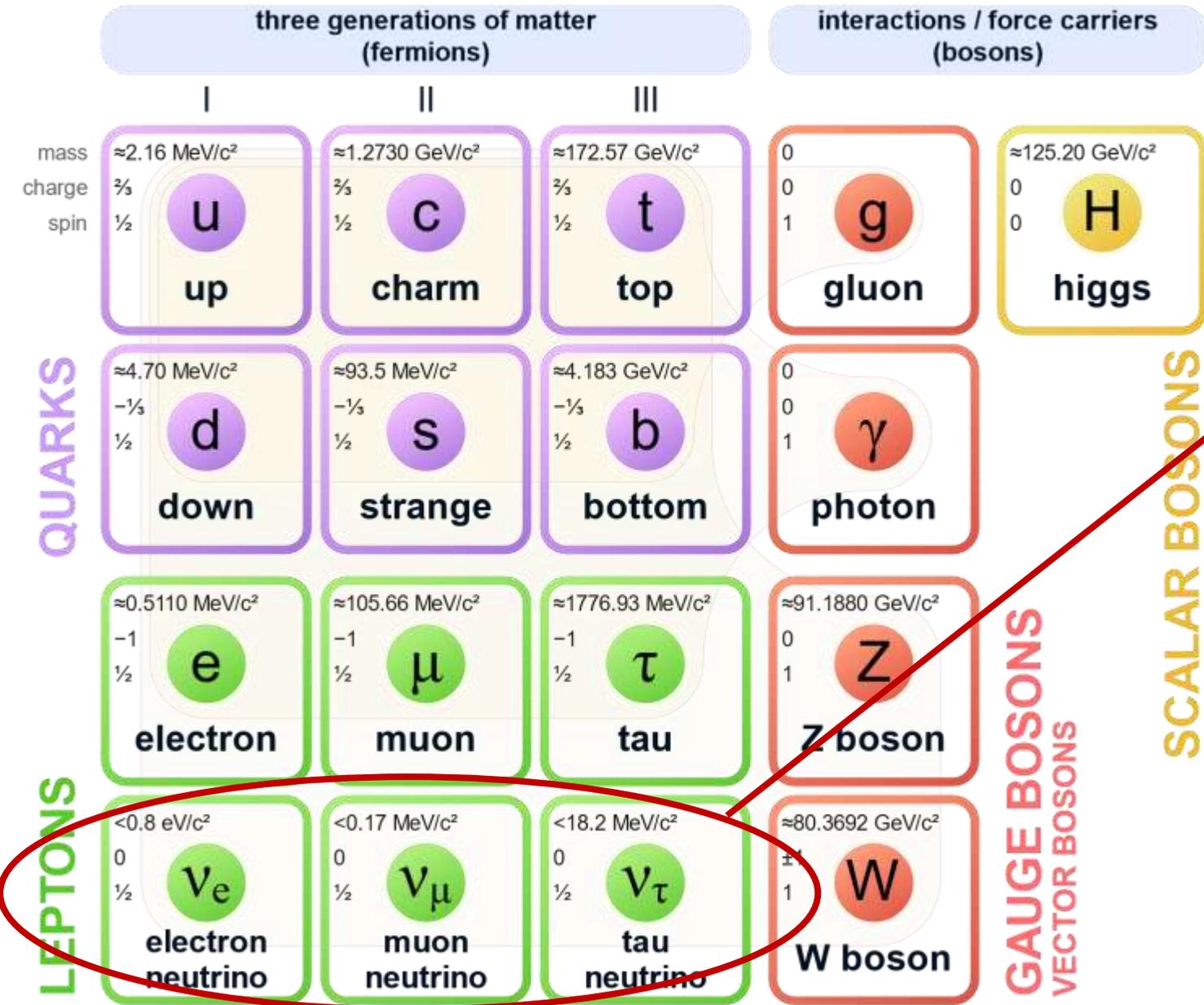
J-Parc 30 high intensity 30 GeV proton beamline  
Tokai, Japan (the T in T2K, and T2HK)

- Quick reminder of neutrinos in standard model
- Discovery of neutrino oscillation
  - Solar neutrino problem (SNO)
  - Atmospheric neutrinos (SuperK)
- Neutrino oscillation physics
- Matter effect on neutrinos
- Overview of a long baseline neutrino experiment

# Acknowledgements

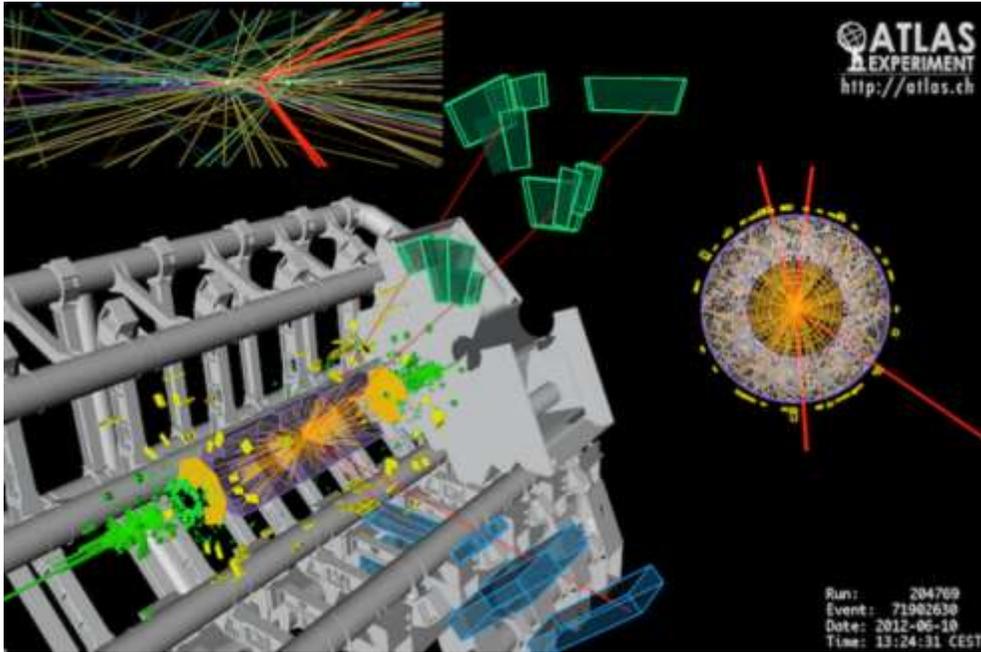
I want to thank all my collaborators, and physics colleagues from whom many of this slides have been acquired for use in the long baseline neutrino oscillation lectures.

# Standard Model of Elementary Particles



- Neutrinos are part of the standard model of particle physics
- Neutral leptons in three generations
- Lightest of the fermions

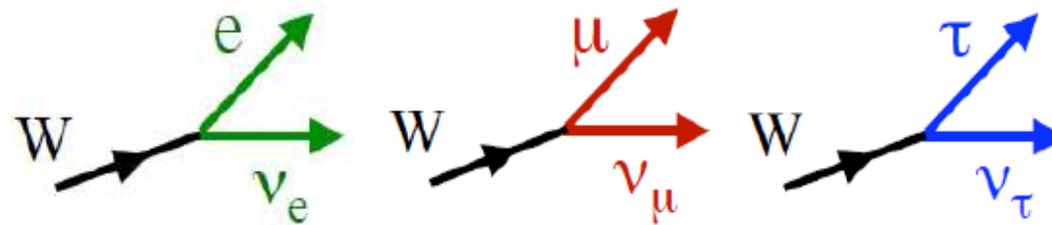
# A quick reminder about neutrino physics



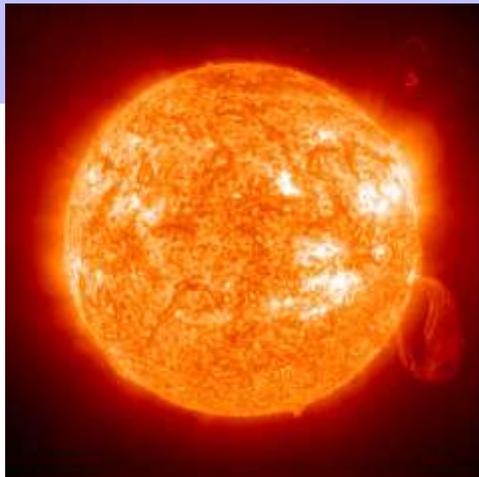
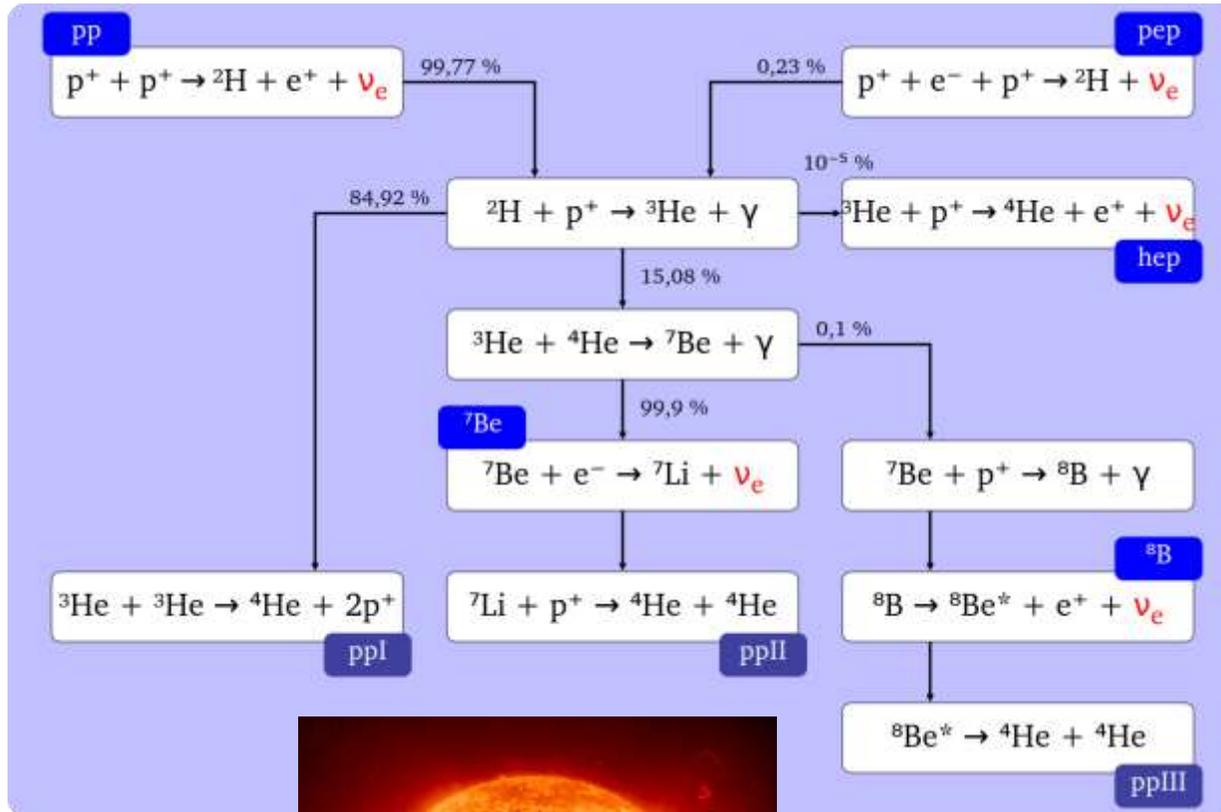
First Higgs particle candidate event in ATLAS (CERN LHC)

- Fundamental constituents of matter:
  - Quarks, charged leptons, neutrinos
- Discovery of Higgs boson at LHC provides strong evidence that quarks and charged leptons get their mass from interaction with the Higgs field
- Many theorists suspect origin of neutrino masses is different
- Discovery of neutrino mass comes from observation of neutrino flavor change (oscillation)

- Define known neutrinos by charged lepton in W boson decays



# Solar neutrinos

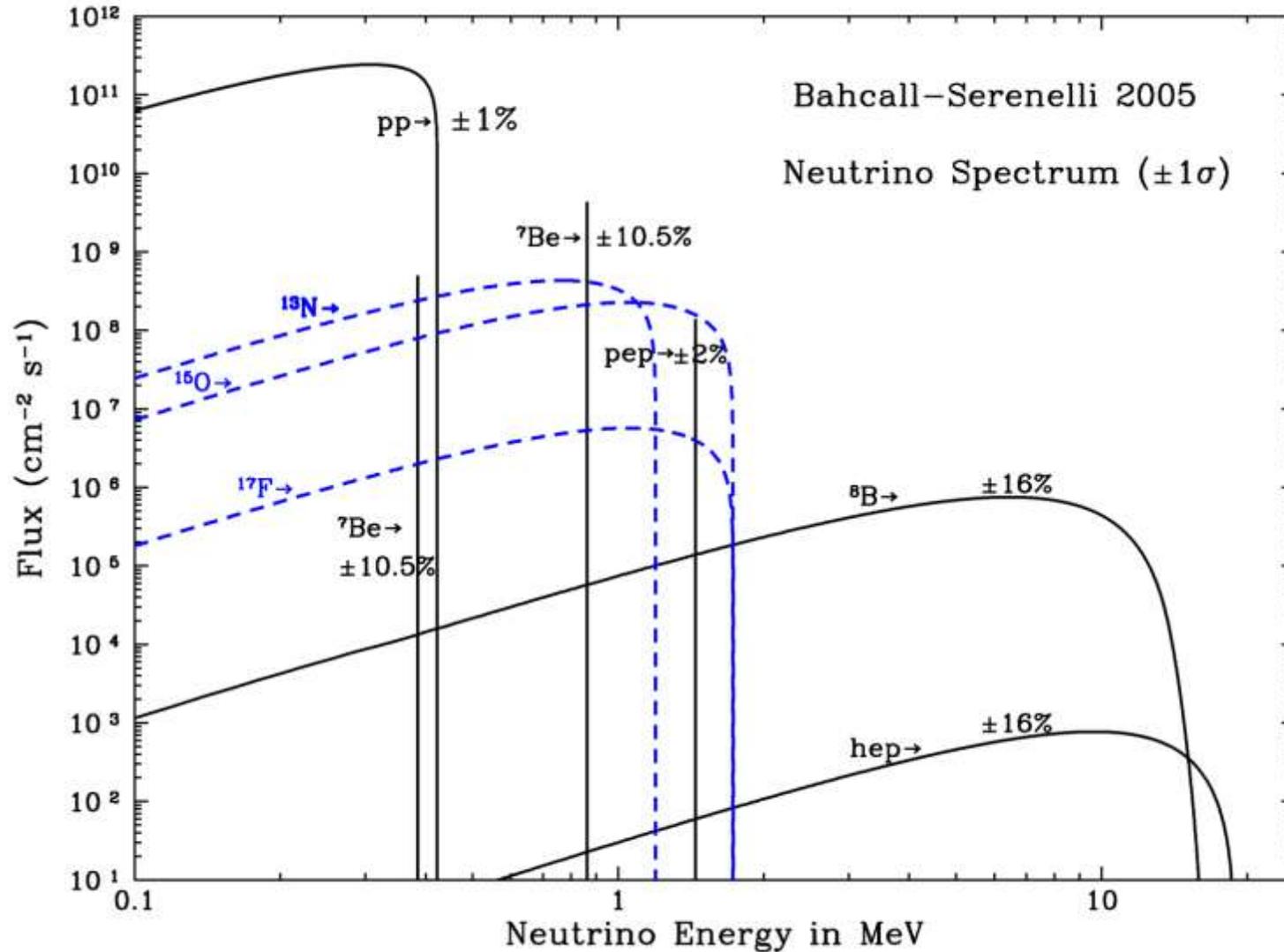


The Sun's fusion reactions produce copious quantities of electron neutrinos!

We know how bright the Sun is and how the fusion reactions work ...

We calculate that 60 billion neutrinos from the Sun pass through your thumbnail every second!

# Neutrinos From the Sun



John Bahcall  
1932-2005

Solar neutrino flux  
predictions

# Looking for solar neutrinos



John Bahcall predicted that solar neutrinos would produce 5.7 atoms/day of Ar in 600 tonne tank of  $C_2Cl_4$ :



Ray Davis (left) went looking for them.

2002 Nobel Prize!

Expected rate:  $5.7 \pm 0.9$  atoms/day  
Measured rate:  $1.9 \pm 0.2$  atoms/day

The Solar Neutrino Problem

Two-thirds of the solar neutrinos were missing!  
Other experiments also see too few solar neutrinos.

Photo from: J. N. Bahcall, "Solar Neutrinos," Proc. 2nd Int. Conference on High-Energy Physics and Nuclear Structure, 1967, pp. 232-255

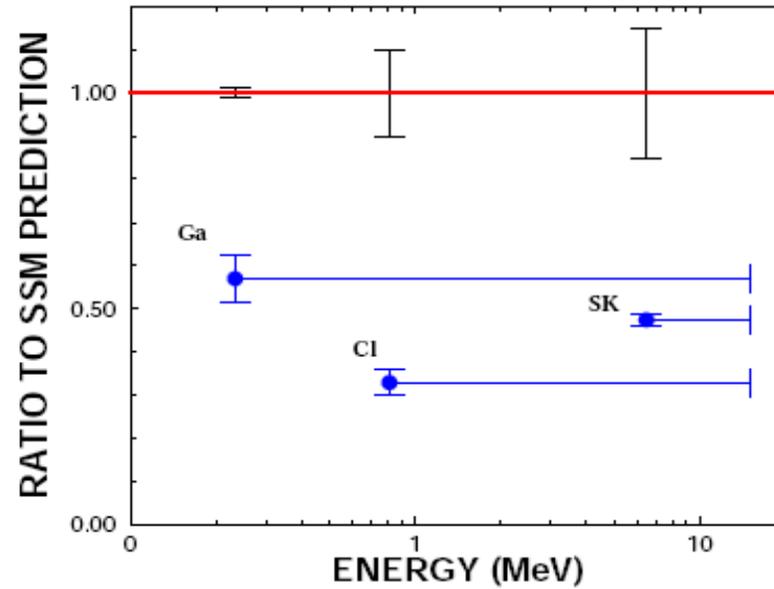
# Solar neutrino problem before SNO

- Radiochemical

- $\nu_e$  interactions convert target nuclei
- Radioactive products extracted and counted after exposure time

- Water Cerenkov

- Real-time detection of scattered atomic  $e^-$ 's
- Mixed CC and NC sensitivity

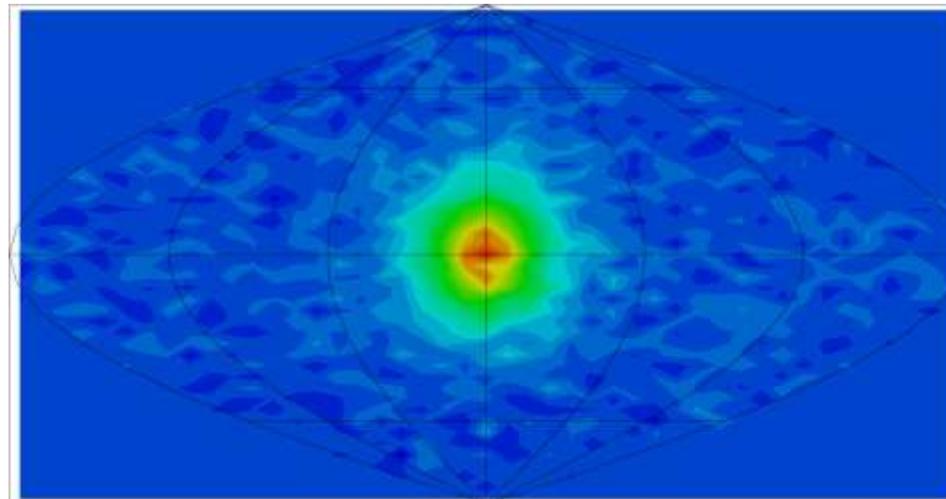


Experiment	Detection Reaction	Threshold	Primary Sources
Homestake	$\nu_e + {}^{37}\text{Cl} \rightarrow e^- + {}^{37}\text{Ar}$	0.8 MeV	${}^7\text{Be}, {}^8\text{B}$
Kamiokande	$\nu_{e,(\mu,\tau)} + e \rightarrow \nu_{e,(\mu,\tau)} + e$	7.3 MeV	${}^8\text{B}$
SAGE, GALLEX/GNO	$\nu_e + {}^{71}\text{Ga} \rightarrow e^- + {}^{71}\text{Ge}$	0.23 MeV	$pp, {}^7\text{Be}, {}^8\text{B}$
Super-K	$\nu_{e,(\mu,\tau)} + e \rightarrow \nu_{e,(\mu,\tau)} + e$	5 MeV	${}^8\text{B}$

# 2002 Nobel Prize

- Went to Ray Davis and Masatoshi Koshiha
- For the discovery of the disappearance of electron neutrinos produced in the sun
- Koshiha was head of the Kamiokande experiment which confirmed Davis' measurement and showed the electron neutrinos came from the sun

Image of neutrinos from the sun



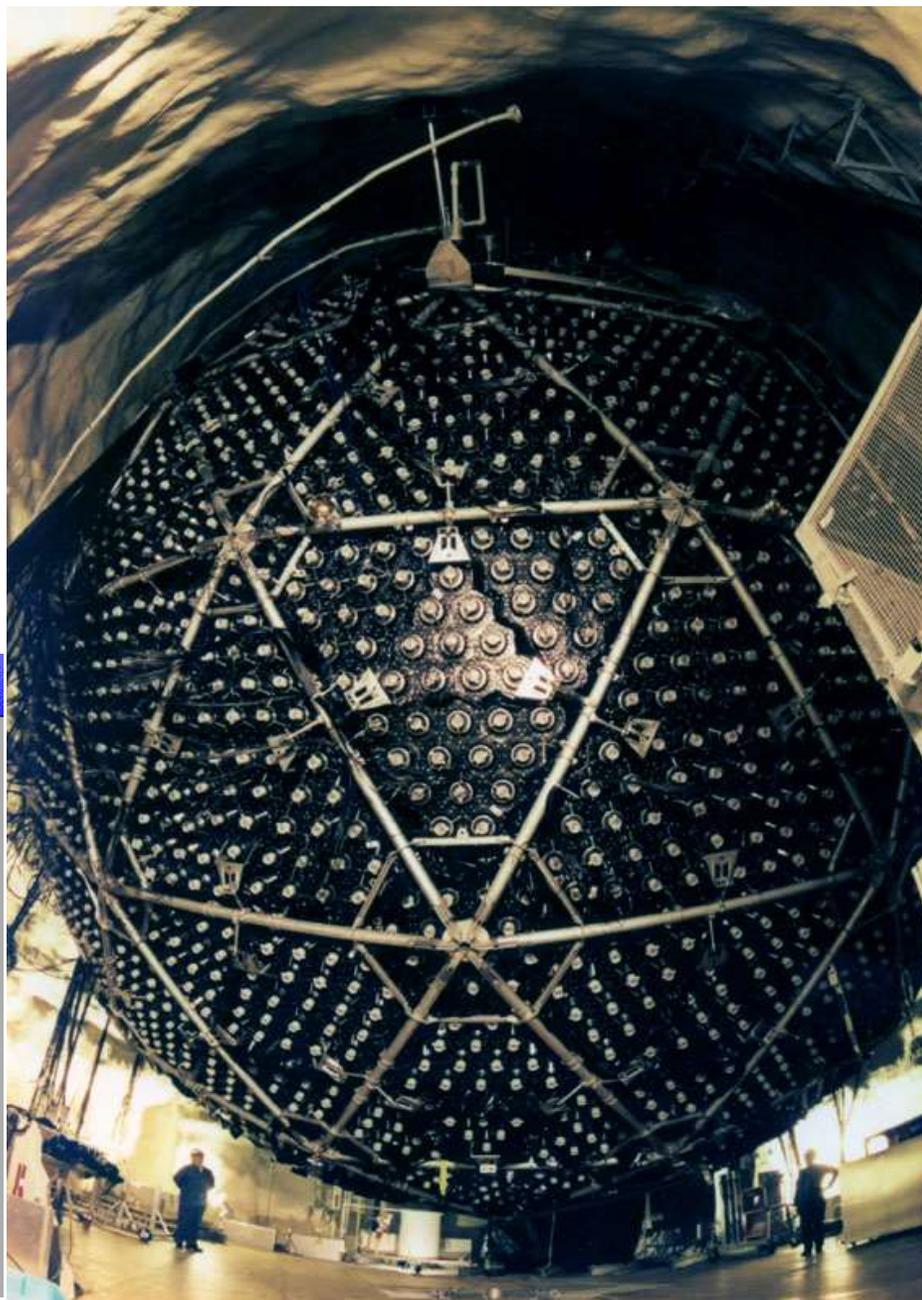
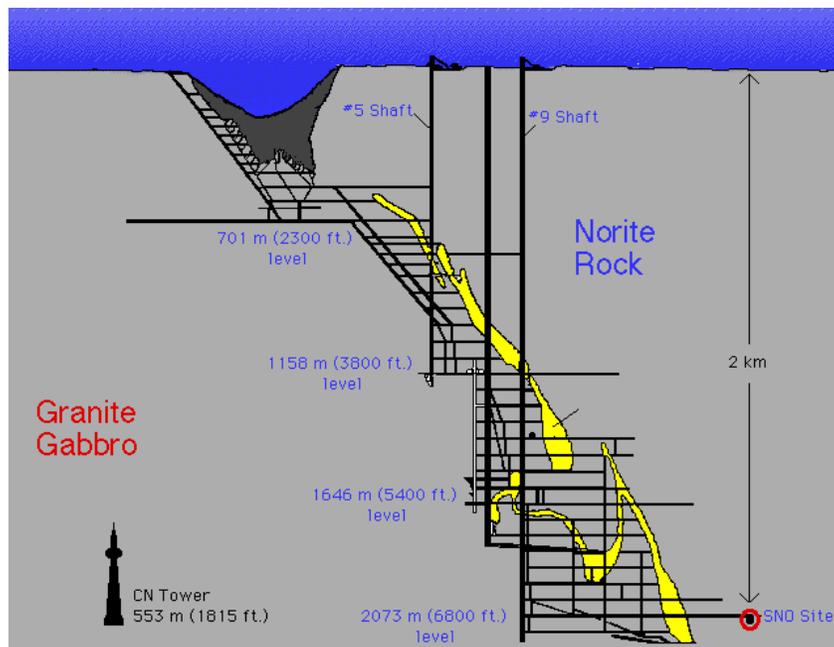
# Why not look for all flavours of neutrinos? Can do that by looking for neutral current

- Dr. Herbert Chen (1942-1987)
  - Professor at UC Irvine
  - proposed using a heavy water Cherenkov detector for this (1984)
  - also started neutrino physics at Los Alamos
- SNO experiment proposed to funding agencies in 1987 after two years of feasibility studies
  - Herb Chen was US PI
  - George Ewan was Canadian PI
  - Art McDonald became US PI at Princeton (now at Queens') after passing of H. Chen

# D2O made by Atomic Energy of Canada for CANDU nuclear reactor moderation

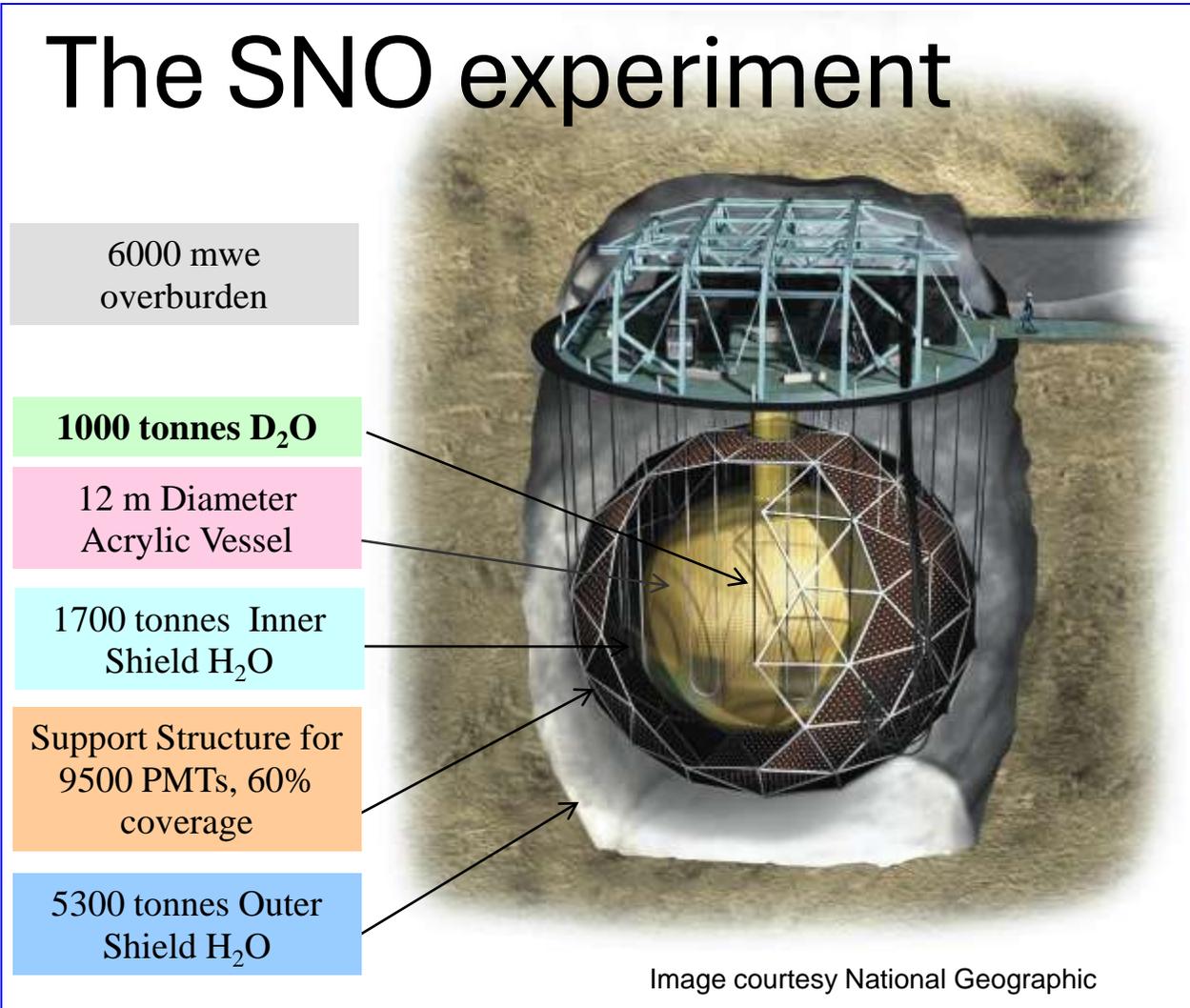


# The SNO detector and cavern

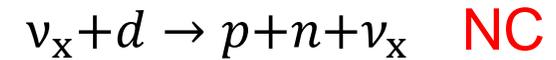
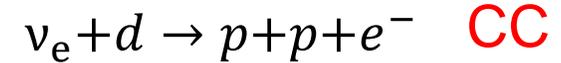
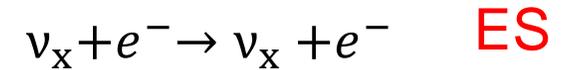


# Resolution of the solar $\nu$ problem

## The SNO experiment



### 3 Reactions:



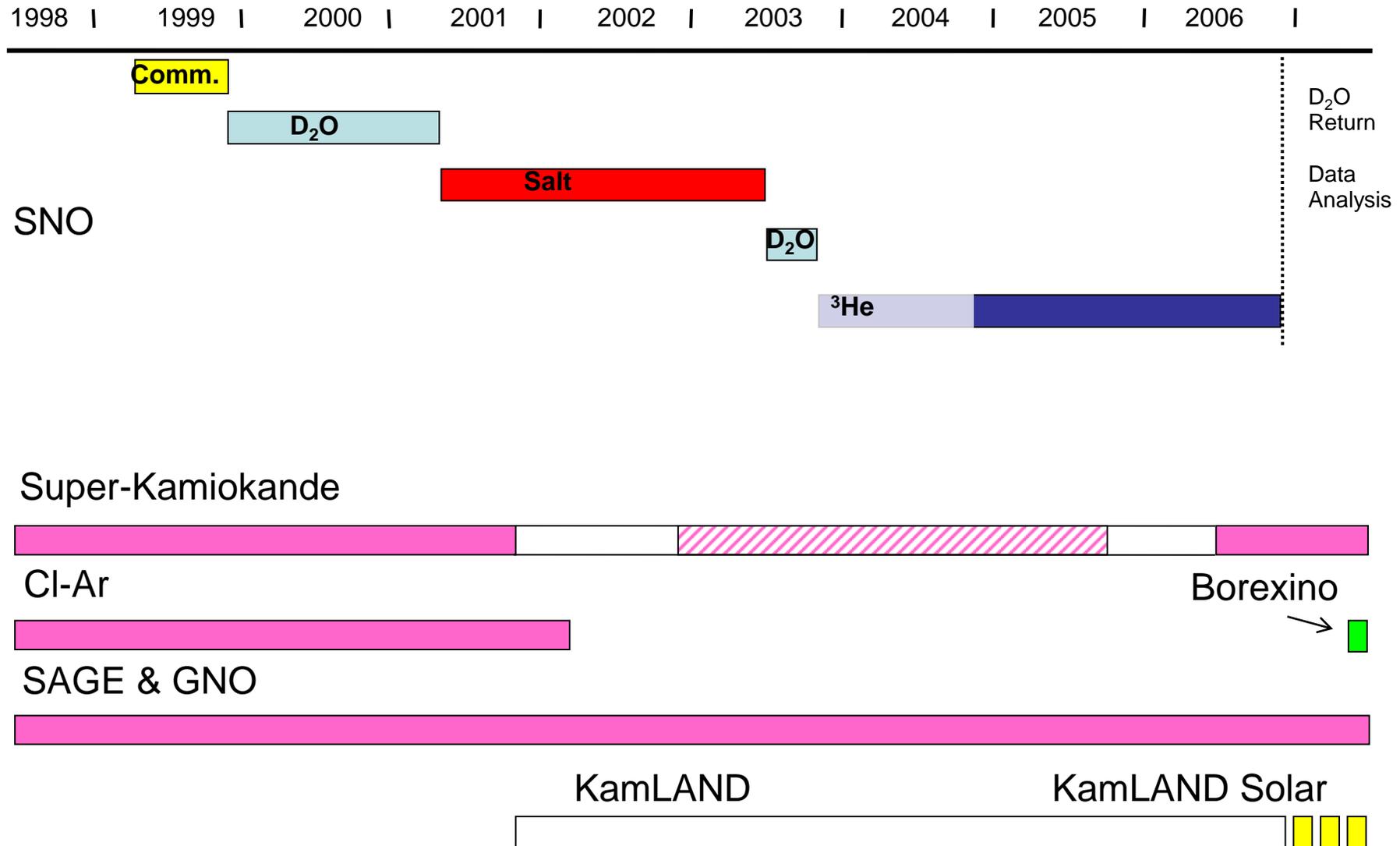
### 3 Phases:

Just D<sub>2</sub>O

D<sub>2</sub>O + 2 tonnes NaCl

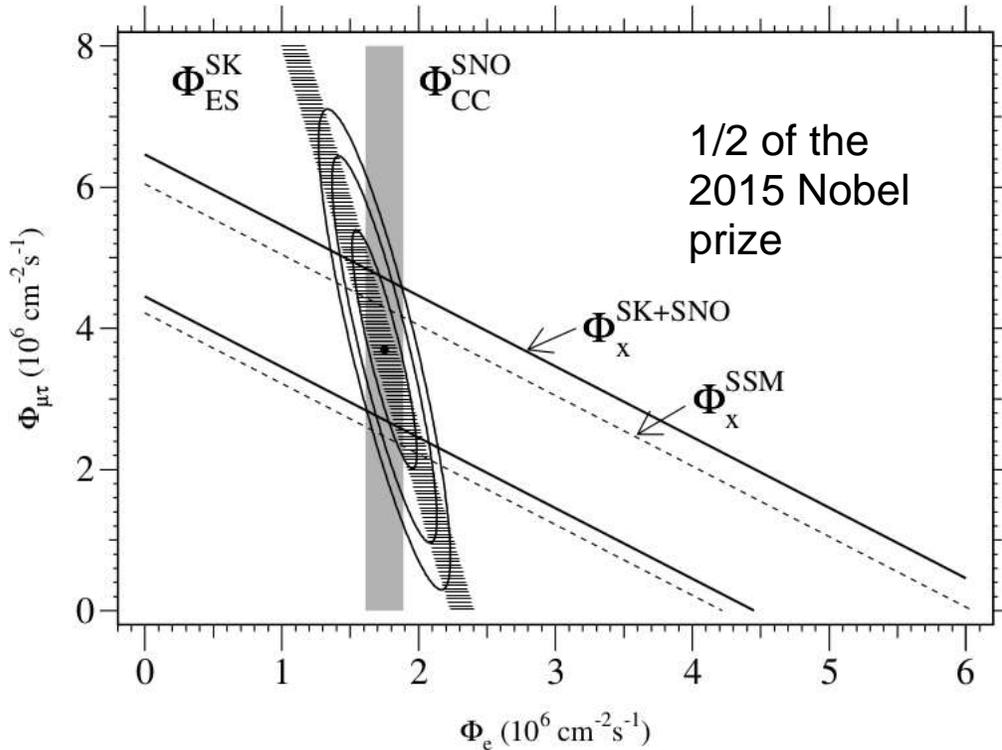
D<sub>2</sub>O + <sup>3</sup>He ("NCDs")

# Timeline of the SNO experiment



# 2001 SNO D2O Phase Result

Phys. Rev. Lett. 87 (2001) 071301  
Solar neutrinos change flavour!



SNO Used heavy water to measure:



From measurements of CC and NC reactions:

$$\frac{\phi_{\nu_e}}{\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau}} = 0.3$$

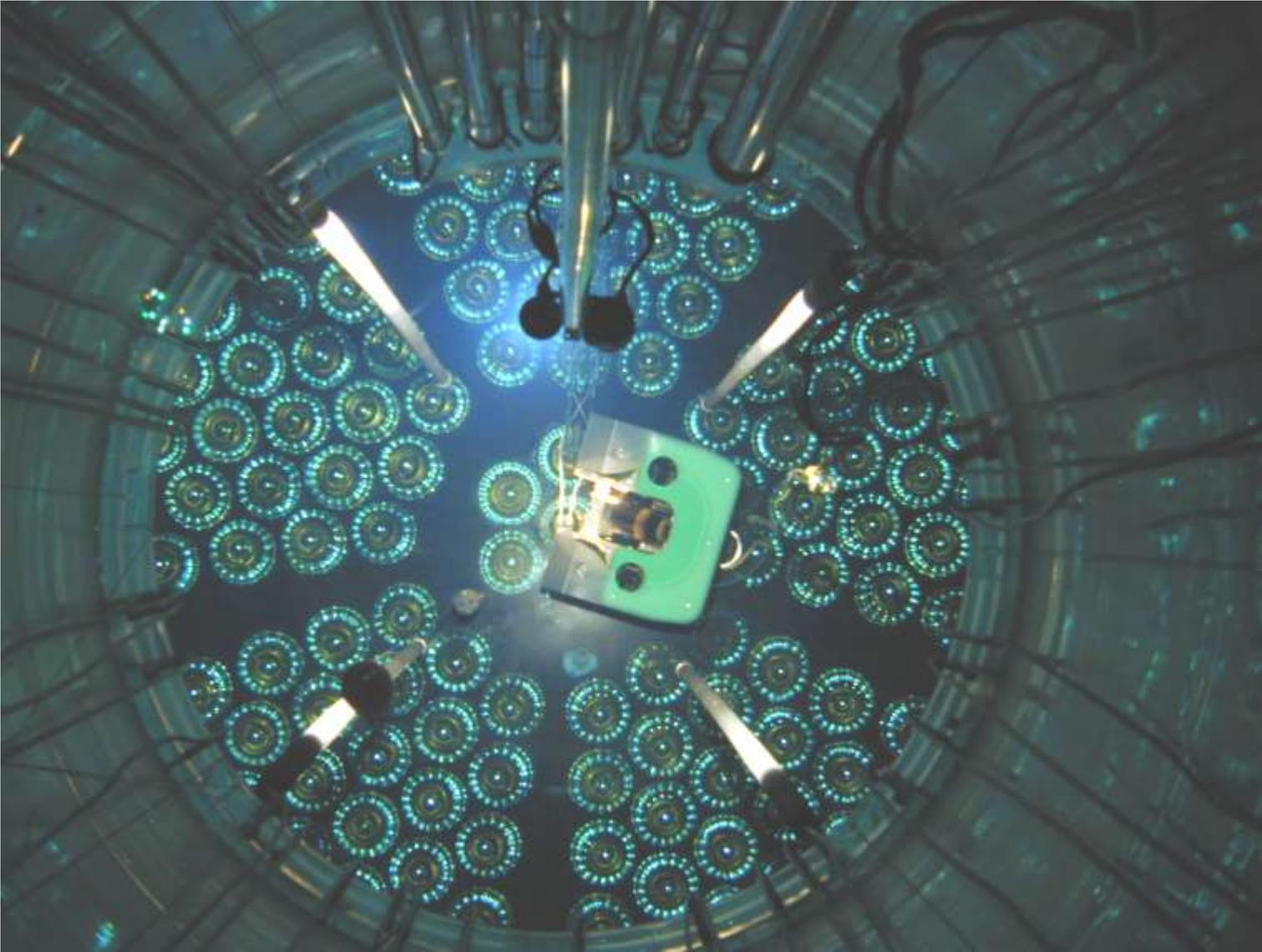
Clearly solar  $\nu$  oscillate

Total flux agrees with Bahcall's prediction

$$\text{SNO: } \phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau} = (5.54 \pm 0.32 \pm 0.35) \times 10^6 \text{ /cm/s}$$

$$\text{Theory: } \phi_{tot} = (5.69 \pm 0.91) \times 10^6 \text{ /cm/s}$$

# SNO Third and Final Phase Neutral Current Detectors (NCD)



# SNO Collaboration during NCD Phase



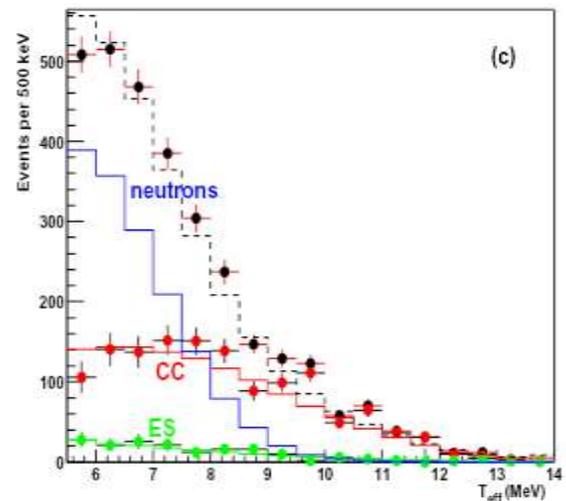
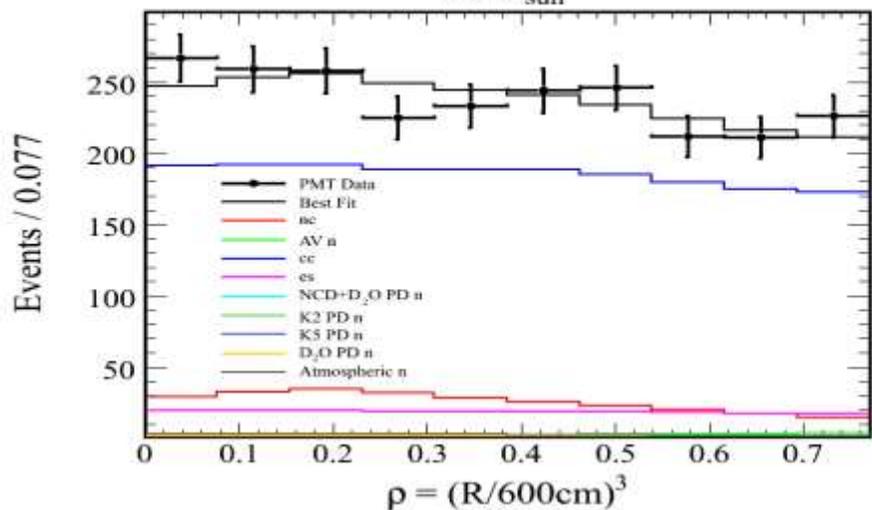
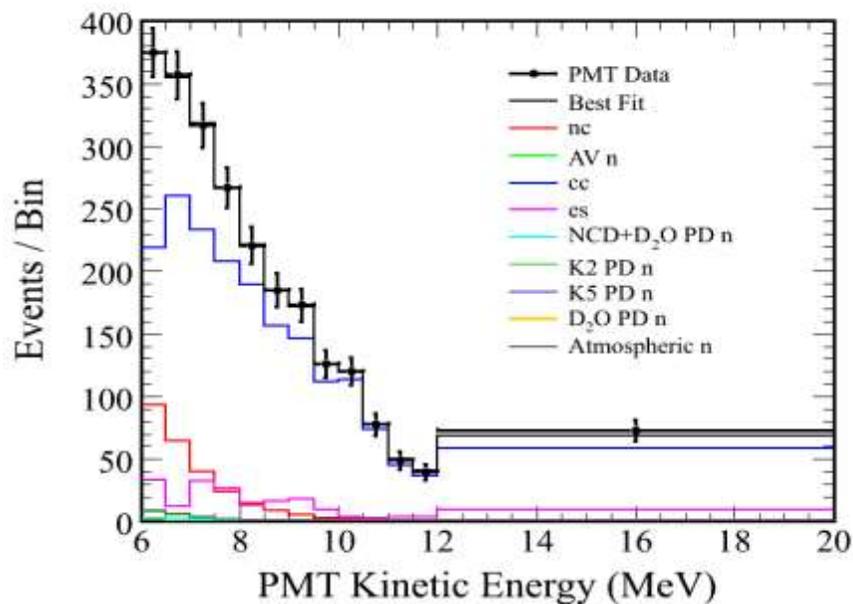
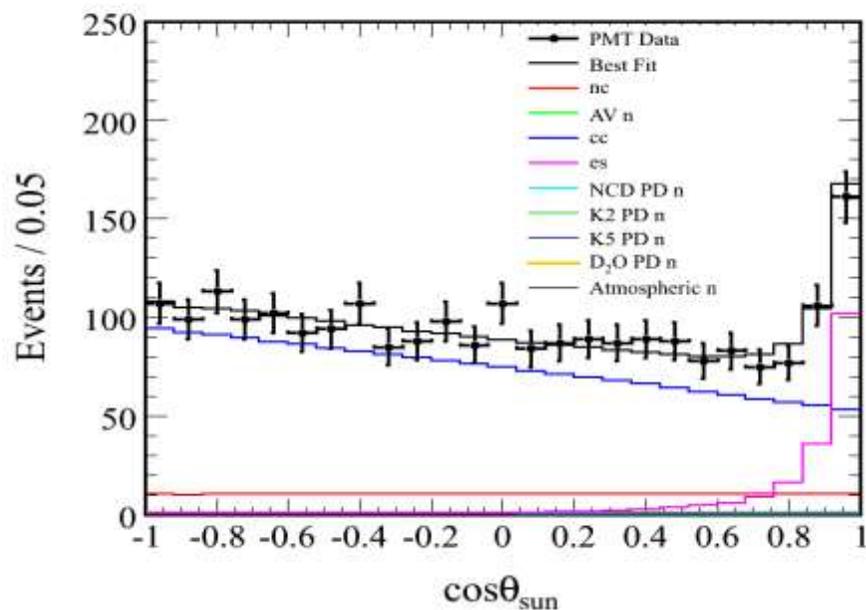
Art McDonald  
Director of SNO experiment



Me!

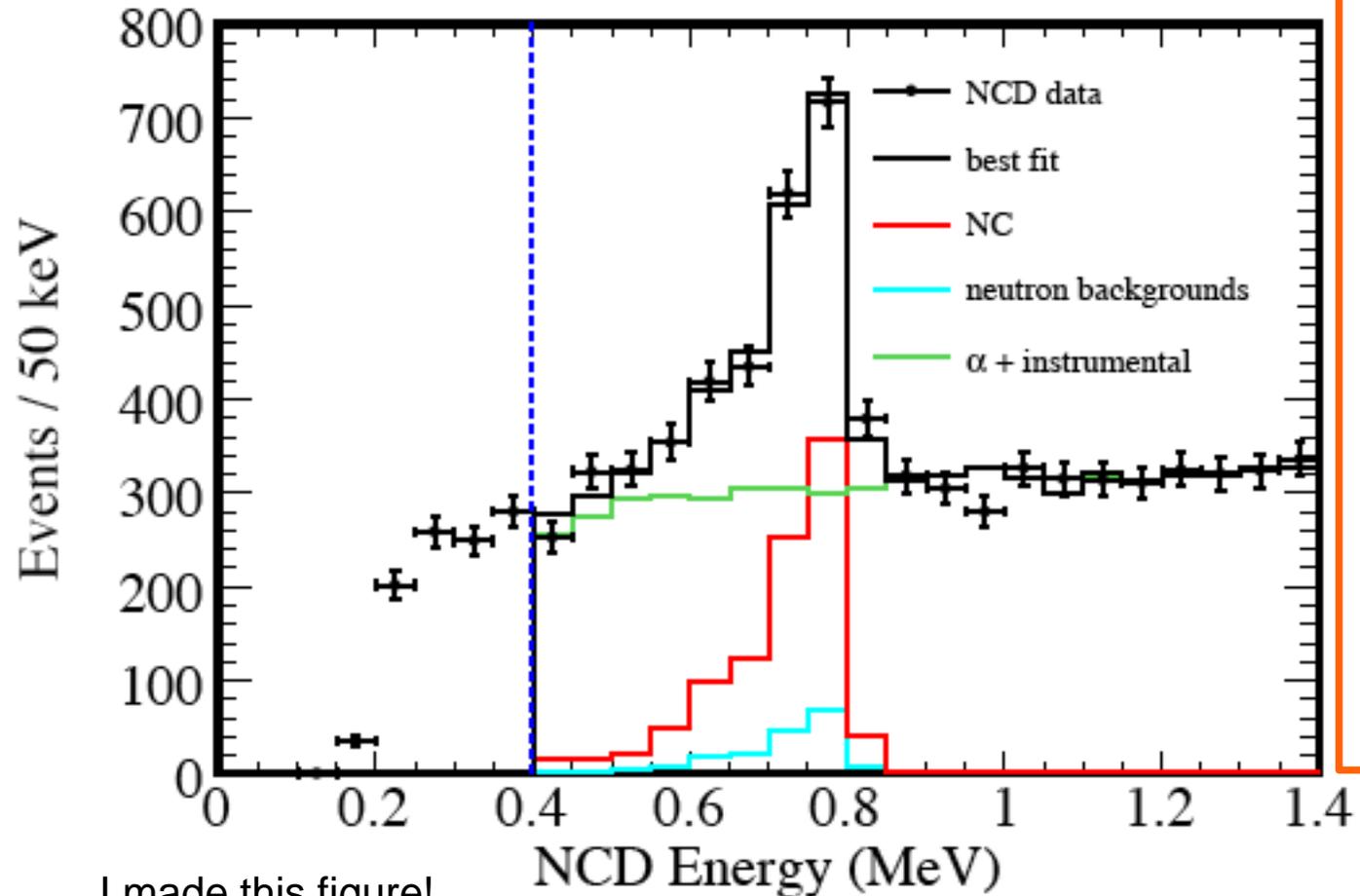


# SNO NCD Phase Heavy Water Data (I made these figures!)



Data from  
salt phase

# Neutrons from solar neutrino interactions



NC Signal:  
 **$983 \pm 77$**

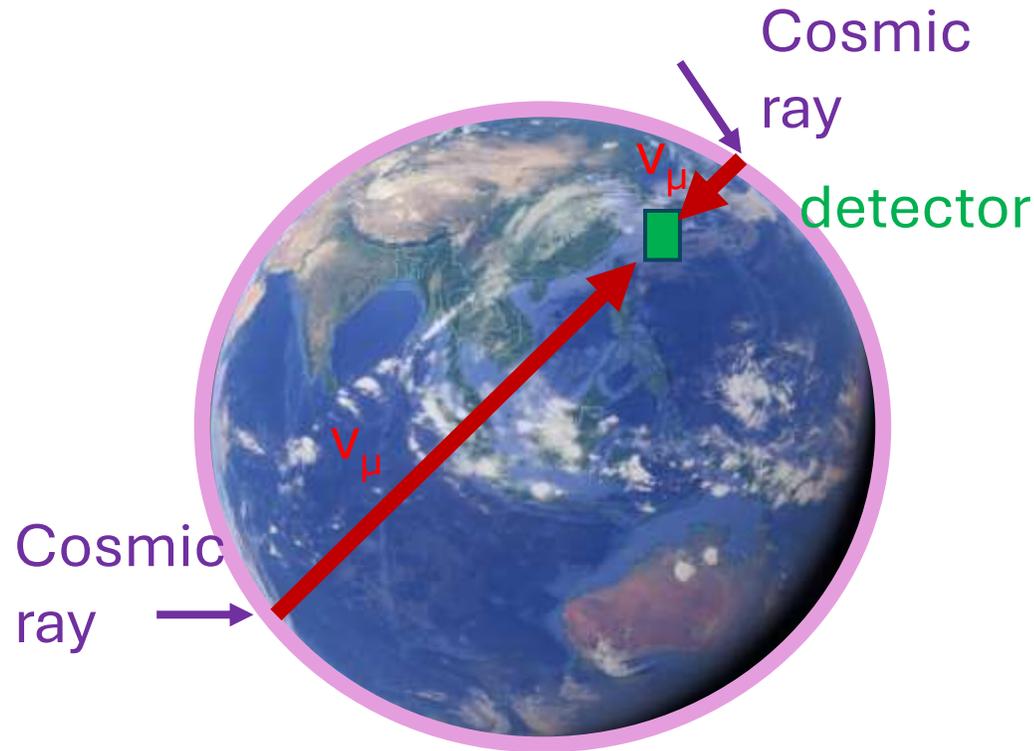
Neutron  
background:  
 **$185 \pm 25$**

Alphas and  
Instrumentals:  
 **$6126 \pm 250$**   
(0.4 to 1.4 MeV)

I made this figure!

# Atmospheric neutrinos

- First detection of neutrino oscillation by Super-Kamiokande (1998)

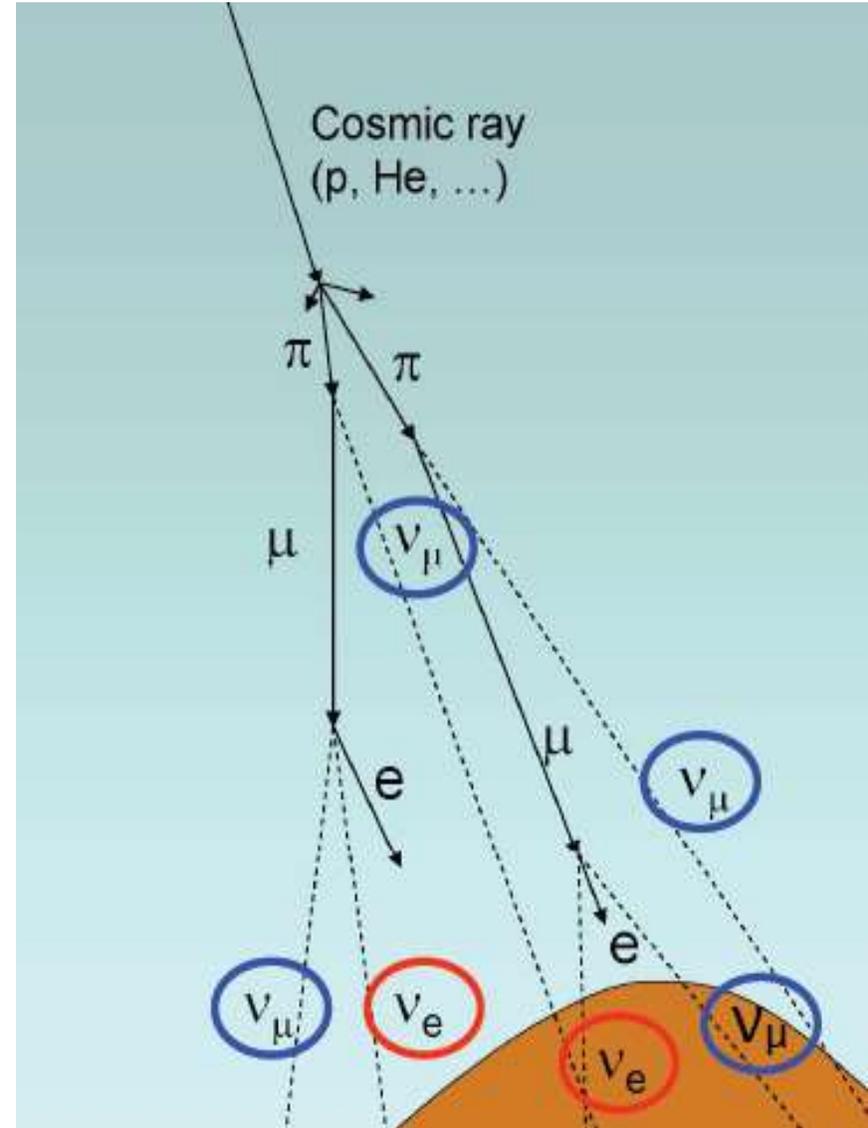


- Isotropy of  $> 2$  GeV cosmic rays +
- Gauss' Law +
- No  $\nu_\mu$  disappearance implies:

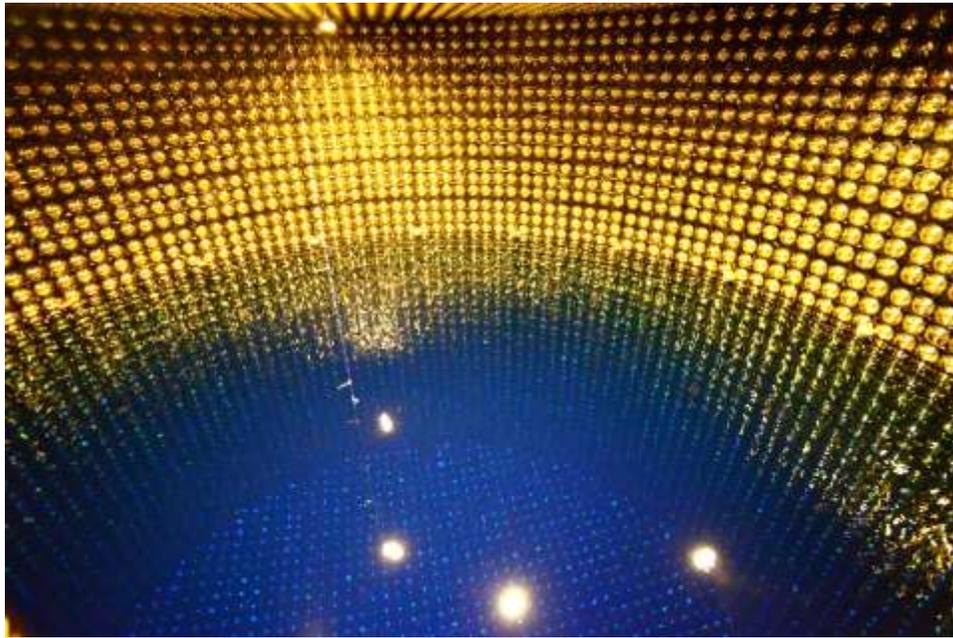
$$\frac{\phi_{\nu_\mu}(\text{upward})}{\phi_{\nu_\mu}(\text{downward})} = 1$$

# Atmospheric neutrinos

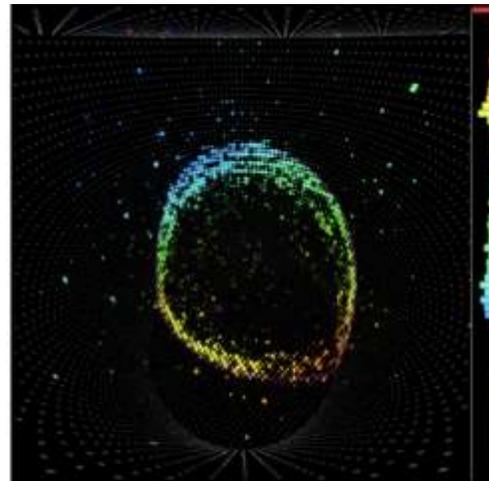
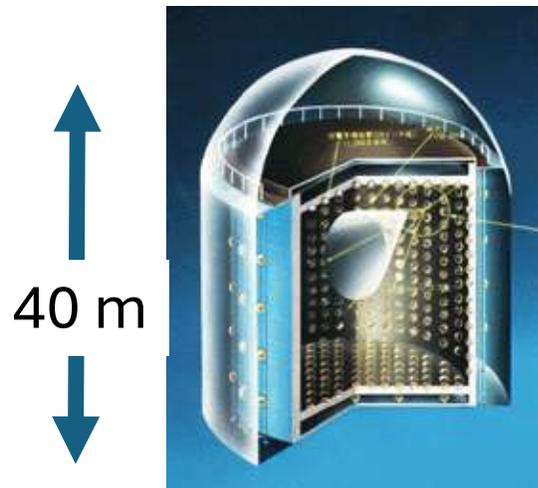
- Cosmic ray showers  $\sim 2\nu_{\mu}:1\nu_e$ 
  - high energy protons from cosmos hitting upper atmosphere produce neutrinos



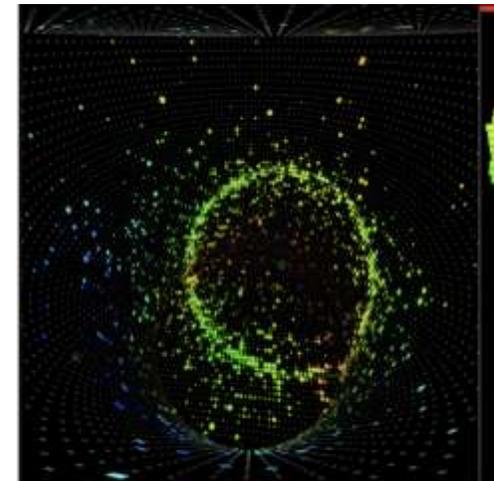
# Super Kamiokande



- 50 kton of water
- Surrounded by 11,000 20" phototubes
- Detects Cerenkov light from  $\mu$  or  $e$
- 1 km under mountain

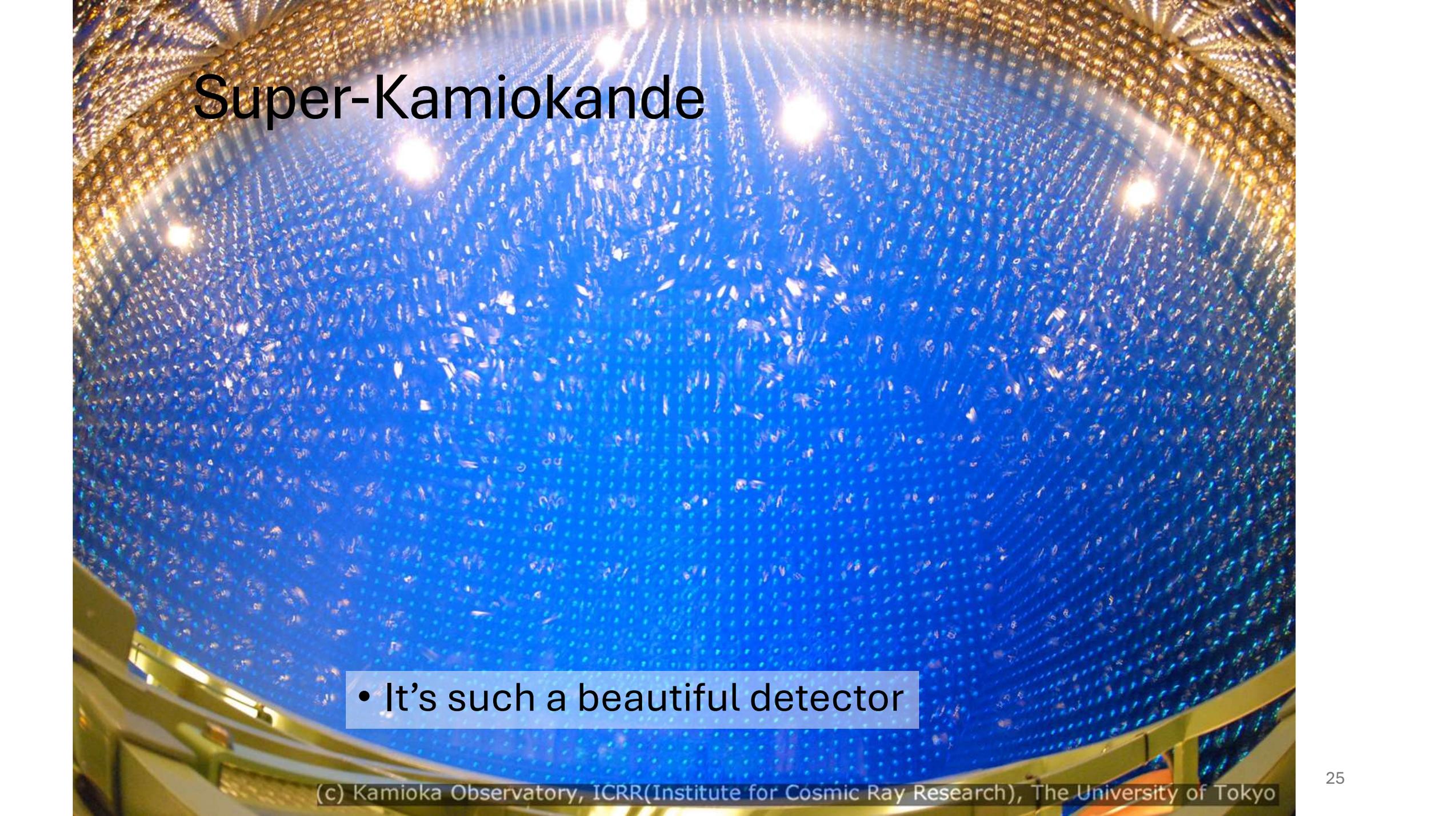


muon-like ( $\nu_\mu$ )



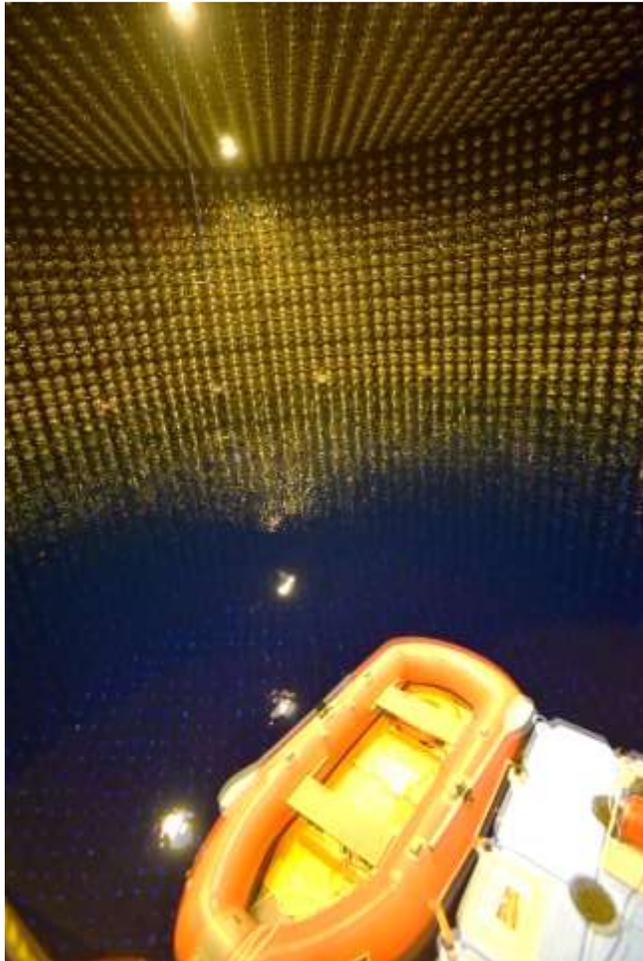
electron-like ( $\nu_e$ )

# Super-Kamiokande

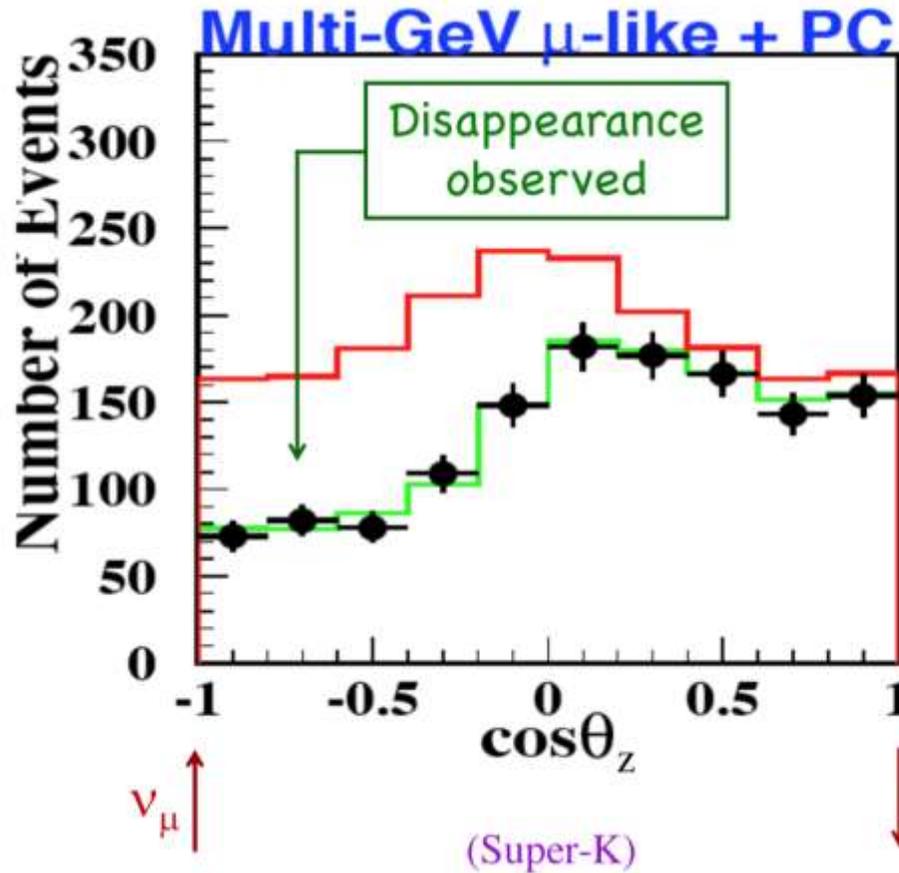


- It's such a beautiful detector

# A few photos from Super-Kamiokande Open tank work (2018)



# Super Kamiokande Atmospheric $\nu$



- For  $E_\nu > 1.3$  GeV, Super-Kamiokande observes

$$\frac{\phi_{\nu_\mu}(\text{upward})}{\phi_{\nu_\mu}(\text{downward})} \cong \frac{1}{2}$$

$$|U_{\mu 3}|^2 \cong \frac{1}{2}$$

L in km  
E in GeV  
 $\Delta m^2$  in eV

We will show  
This later!

$$P(\nu_\mu \rightarrow \nu_\mu) \cong \underbrace{1}_{1/2} - \underbrace{4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2)}_1 \underbrace{\sin^2 \left[ 1.27 \Delta m_{\text{atm}}^2 \frac{L}{E} \right]}_{1/2}$$

# Neutrino Mixing Two-Flavor Model

$$|\nu_e\rangle = \cos \theta |\nu_1\rangle + \sin \theta |\nu_2\rangle$$

$$|\nu_\mu\rangle = -\sin \theta |\nu_1\rangle + \cos \theta |\nu_2\rangle$$

Each term evolves with a phase factor of  $e^{i(px-Et)}$

If  $m_1 \neq m_2$ , then arguments of exponential will be different! For example, if we consider  $p$  to be fixed, then

$$E = \sqrt{p^2 + m^2} = p\sqrt{1 + m^2/p^2} \approx p + m^2/(2p)$$

As neutrino propagates, a phase difference develops between terms

$$|\nu(t)\rangle \propto \cos \theta |\nu_1\rangle + e^{i\phi} \sin \theta |\nu_2\rangle$$

with

$$\phi = \left( \frac{m_1^2}{2p} - \frac{m_2^2}{2p} \right) t$$

# Neutrino Oscillations

The formula for a neutrino changing into a different kind :

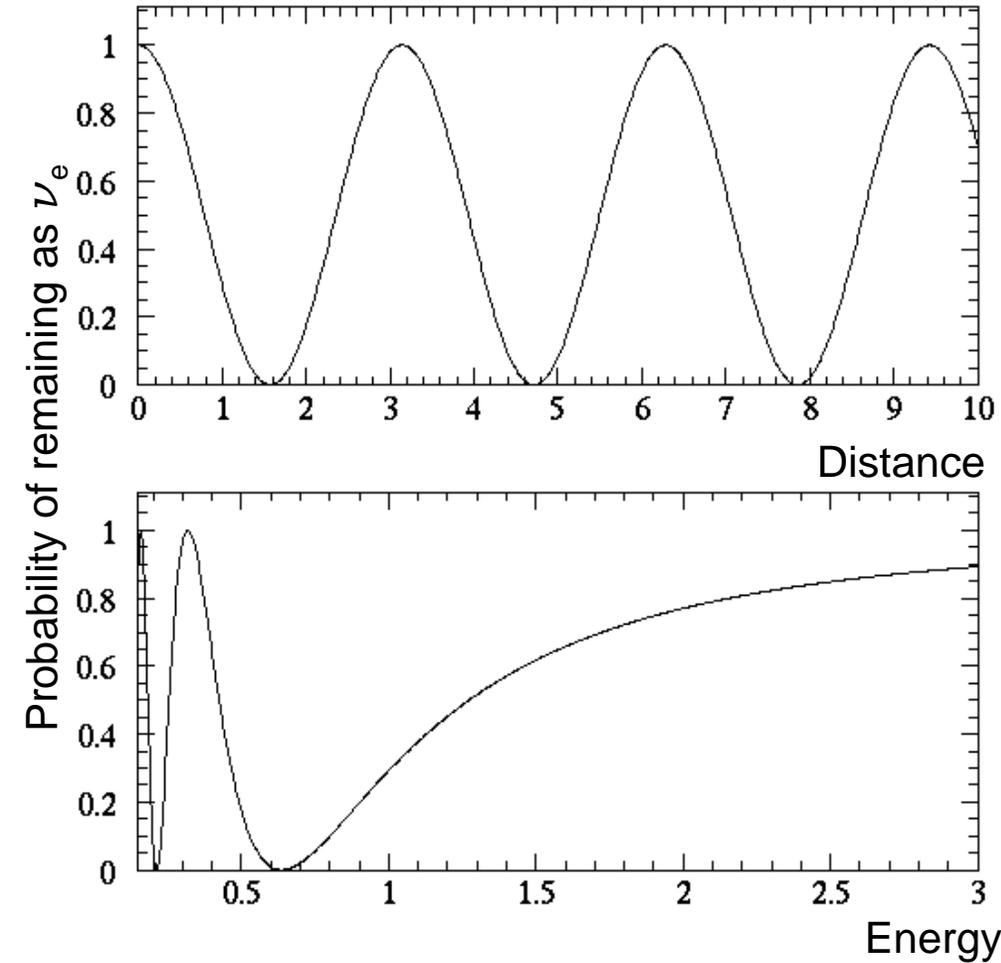
$$P(\nu_e \rightarrow \nu_\mu) = |\langle \nu_\mu | \nu(t) \rangle|^2 = \sin^2 2\theta \sin^2 \left( \frac{1.27 \Delta m^2 L}{E} \right)$$

$\sin^2 2\theta$  = mixing angle that controls the amplitude of the oscillation

$$\Delta m^2 = (\text{mass}_2)^2 - (\text{mass}_1)^2$$

L = distance neutrino travelled

E = energy of neutrino

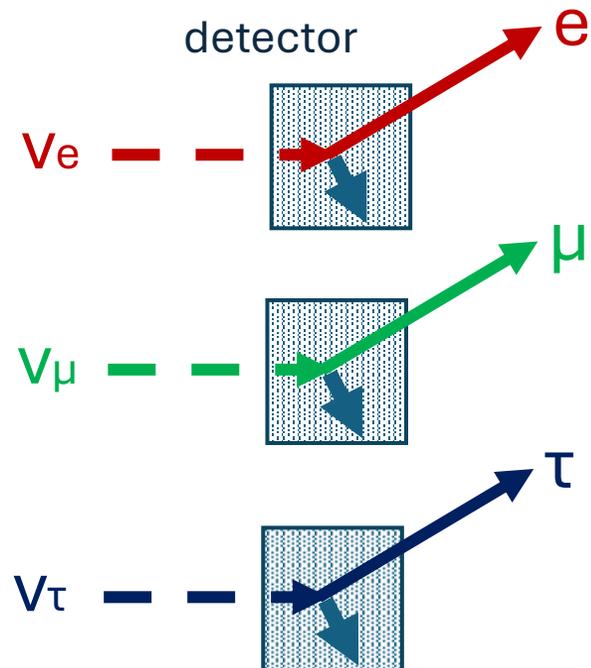


Slide from S. Oser  
Ca. 2007

Above formula for  $\Delta m^2$  in  $\text{eV}^2$ , L in km, E in GeV

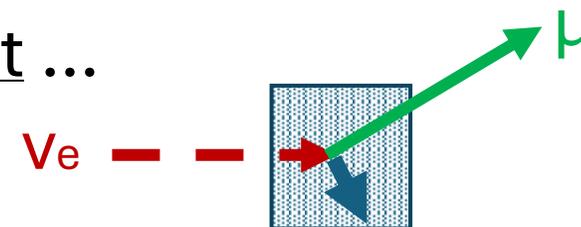
# Neutrino flavor conservation

Allowed interactions:



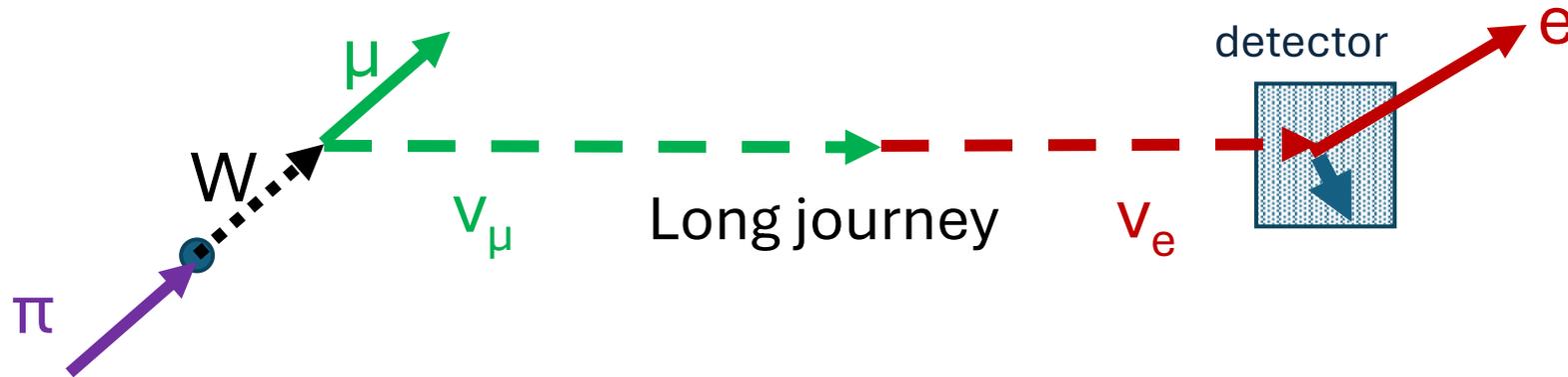
- When a neutrino of a given flavor interacts, the charged lepton flavor matches that of the neutrino
- First evidence in 1962 -- Leon Lederman, Mel Schwartz, Jack Steinberger at BNL
- Observed spontaneous appearance in a detector near accelerator production of muon neutrinos

But not ...



# Neutrino flavor change (oscillation)

- If neutrinos have mass, and leptons mix we can have:



- Give a neutrino time to change flavor and you can have  $\nu_\mu \rightarrow \nu_e$
- The last 25 years have brought us compelling evidence that such flavor changes actually occur

# Flavor change requires lepton mixing

- The neutrinos of definite flavor  $\nu_\alpha$  ( $W \rightarrow e \nu_e$ ,  $W \rightarrow \mu \nu_\mu$ ,  $W \rightarrow \tau \nu_\tau$ ) do not have a definite mass – they are superpositions of mass states
- Must be super-positions of the mass eigenstates  $\nu_i$

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle$$

Neutrino flavor  
 $\alpha = e, \mu, \tau$

PMNS  
Leptonic mixing matrix

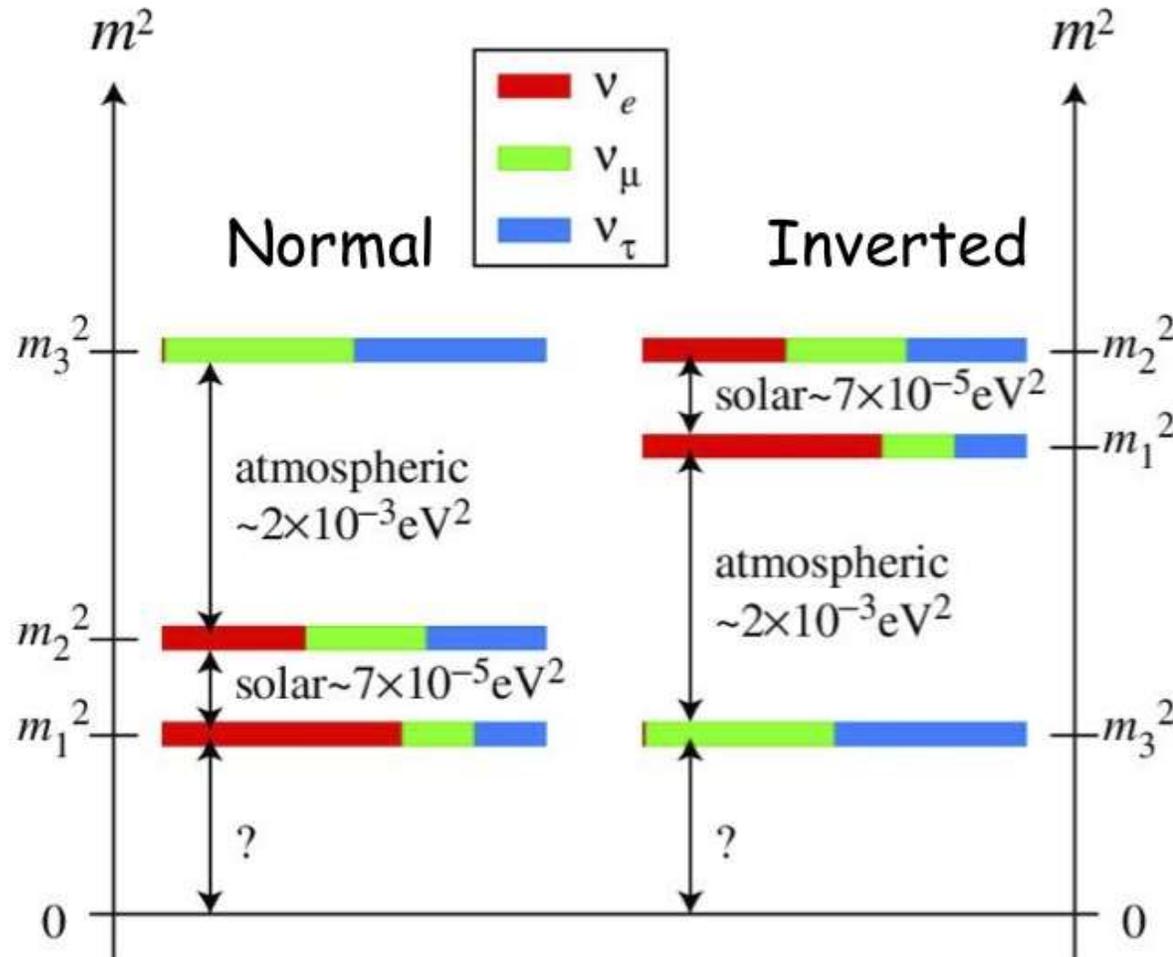
Neutrino of  
mass  $m_i$

- As far as we know U is unitary, then flavor fraction is:

$$f_i = |U_{\alpha i}|^2$$

- If there are only three mass eigenstates, U is a 3 x 3 matrix

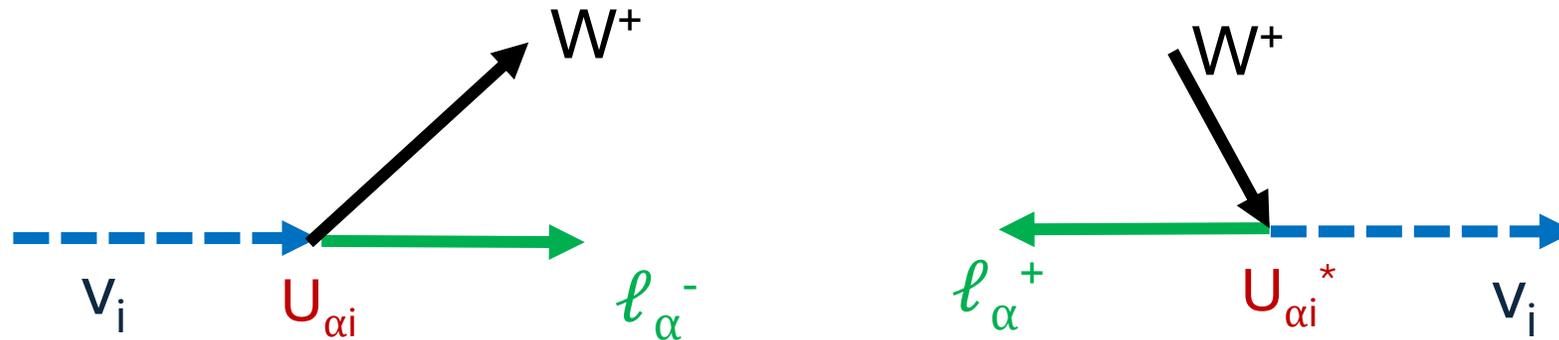
# Neutrino flavor change requires neutrino masses



- There must be a mass spectrum of neutrino eigenstates
- Oscillation experiments have measured two mass-squared differences
  - Don't know sign of larger one
- Cosmology:  $\sum_i m(\nu_i) < 0.17 \text{ eV}$
- Tritium  $\beta$  decay:  $m_{\nu_e} < 0.8 \text{ eV}$  (Katrin)
- Oscillations:

$$m(\text{heaviest}) > \sqrt{\Delta m_{\text{big}}^2} > 0.05 \text{ eV}$$

# Standard Model Lagrangian with $\nu$ Mixing



- Lepton mixing is easily incorporated in the Standard Model description of  $W \rightarrow \ell \nu$  interaction, Lagrangian is

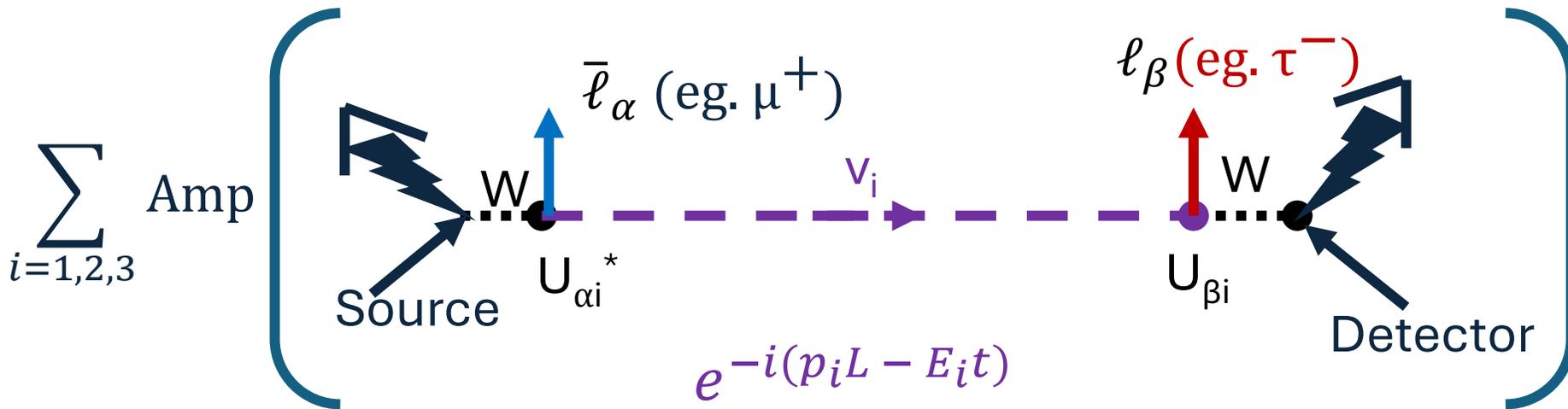
Semi-weak  
Coupling  $g$

$$\mathcal{L} = -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\pi \\ i=1,2,3}} \left( \bar{\ell}_{L\alpha} \gamma^\lambda U_{\alpha i} \nu_{Li} W_\lambda^- + \bar{\nu}_{Li} \gamma^\lambda U_{\alpha i}^* \ell_{L\alpha} W_\lambda^+ \right)$$

- The SM conserves lepton number  $L$ :  $L(\nu) = L(\ell^-) = -L(\bar{\nu}) = -L(\ell^+) = 1$

# Neutrino oscillation ( $\nu_\alpha \rightarrow \nu_\beta$ )

- Calculation of interaction amplitude



Plane wave treatment:

Neutrino: Momentum  $p_i$  Energy  $E_i$   
 Coordinates of source  $(0,0)$   
 Detector at  $(t,L)$

# Only coherent energy neutrinos oscillate

- Averaged over time:

$$\langle e^{-i(E_1 t - E_2 t)} \rangle$$

- is zero, unless  $E_2 = E_1$
- Only neutrino mass eigenstates with common energy  $E$  are coherent
- For each mass eigenstate  $\nu_i$

$$p_i = \sqrt{E^2 - m_i^2} \cong E - \frac{m_i^2}{2E}$$

- Plane-wave factor is:

$$e^{i(p_i L - Et)} \cong e^{-i\left\{\left(E - \frac{m_i^2}{2E}\right)L - Et\right\}} = e^{iE(L-t)} e^{-im_i^2 \frac{L}{2E}}$$

Irrelevant overall  
phase factor



# Probability of Oscillation in vacuum

$$\text{Amp}[\nu_\alpha \rightarrow \nu_\beta] = \sum_{i=1,2,3} U_{\alpha i}^* e^{-im_i^2 \frac{L}{2E}} U_{\beta i}$$

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) &= |\text{Amp}[\nu_\alpha \rightarrow \nu_\beta]|^2 \\ &= \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2 \left( \Delta m_{ij}^2 \frac{L}{4E} \right) \\ &\quad + 2 \sum_{i>j} \text{Im}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin \left( \Delta m_{ij}^2 \frac{L}{2E} \right) \end{aligned}$$

Where  $\Delta m_{ij}^2 = m_i^2 - m_j^2$

→  $\nu$  flavor change implies neutrino mass!

# Neutrinos and anti-neutrinos

$$[\bar{\nu}_\alpha(RH) \rightarrow \bar{\nu}_\beta(RH)] = CP[\nu_\alpha(LH) \rightarrow \nu_\beta(LH)]$$

- A difference between the probabilities of neutrino and anti-neutrino oscillations in vacuum would be a leptonic violation of Charge Parity (CP) violation

$$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2 \left( \Delta m_{ij}^2 \frac{L}{4E} \right) \\ \pm 2 \sum_{i>j} \text{Im}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin \left( \Delta m_{ij}^2 \frac{L}{2E} \right)$$

- In neutrino oscillation, CP violation comes from complex phases in PMNS matrix U

# The PMNS matrix U

$$U = \begin{matrix} & \nu_1 & \nu_2 & \nu_3 \\ e & U_{e1} & U_{e2} & U_{e3} \\ \mu & U_{\mu1} & U_{\mu2} & U_{\mu3} \\ \tau & U_{\tau1} & U_{\tau2} & U_{\tau3} \end{matrix}$$

Majorana phases  
don't affect oscillation  
Or introduce CP-violation

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$c_{ij} = \cos \theta_{ij} \quad s_{ij} = \sin \theta_{ij}$$

- The phase  $\delta \neq 0$  violates CP, and leads to:

$$P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$$

- Note crucial value  $\sin\theta_{13}$

# The oscillation parameters ( $1\sigma$ )

$\theta_{12}$	[ 32.46 – 34.45 ] degrees
$\theta_{23}$	[ 40.1 – 57.7 ] degrees
$\theta_{13}$	[ 7.27 – 9.10 ] degrees
$\delta_{CP}$	180 – 230 degrees
$\Delta m_{21}^2$	$(7.35 – 7.77) \times 10^{-5} \text{ eV}^2$
$\Delta m_{31}^2$	$\left\{ \begin{array}{l} +(2.48 \text{ to } 2.53) \times 10^{-3} \text{ eV}^2 \text{ Normal order} \\ -(2.39 \text{ to } 2.44) \times 10^{-3} \text{ eV}^2 \text{ Inverted order} \end{array} \right.$

From global fit of: Valencia PreNu 2024 (Presented at Neutrino 2024 Conference)

## Leptonic CP Violation

Mixing matrix is substantially off-diagonal

$$|U_{\text{PMNS}}| \approx \begin{pmatrix} 0.80 - 0.85 & 0.51 - 0.58 & 0.14 - 0.16 \\ 0.23 - 0.52 & 0.44 - 0.70 & 0.61 - 0.79 \\ 0.25 - 0.53 & 0.46 - 0.71 & 0.59 - 0.78 \end{pmatrix}$$

Mixing matrix is much more diagonal

$$|U_{\text{CKM}}| \approx \begin{pmatrix} 0.974 & 0.225 & 0.004 \\ 0.225 & 0.973 & 0.04 \\ 0.009 & 0.04 & 0.999 \end{pmatrix}$$

Jarlskog invariant:  
Scale of maximum CP-violating effect from the mixing

$$J = \sin(2\theta_{12})\sin(2\theta_{13})\sin(2\theta_{23})\cos(\theta_{13})\sin(\delta)/8$$

**Lepton sector:  $0 \leq |J_{\text{PMNS}}| \leq 0.03$**

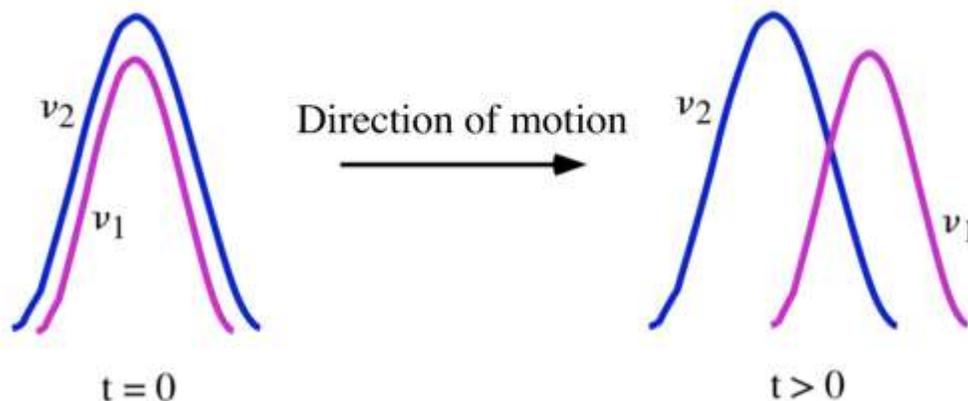
**Quark sector:  $J_{\text{CKM}} \leq 0.00003$**

**Is CPV in  $U_{\text{PMNS}}$  related to the Baryon Asymmetry of the Universe**

Leptogenesis: CP-violating process created matter-antimatter asymmetry in leptons that was transferred to baryons in early universe

# Wave packet treatment of oscillation

- Probability  $\nu$  oscillates depends on L between source and detection
- A plane wave has definite momentum p
- Heisenberg:  $\Delta x \Delta p \geq \frac{\hbar}{2}$
- If we know precisely the momentum with which neutrino is born, we know nothing about where it was born
- Each  $\nu$  eigenstate is wave packet – suppose  $\nu_2$  is heavier than  $\nu_1$



Eventually wave packets  
Will separate (no more  
Oscillation)

# How soon do wave packets separate??

- For accelerator with  $E_\nu \sim 1 \text{ GeV}$ 
  - Wave packet width = length of pion decay region
  - Bigger  $\Delta m_{32}^2 = 2.4 \times 10^{-3} \text{ eV}^2$ , wave packet sep. in
  - $10^{20} \text{ km}$  – safely ignored for experiment on earth
- For supernova  $\nu$  (SN1987A)  $E_\nu \sim 10 \text{ MeV}$ 
  - Wave packet width = inter-nucleon spacing in star
  - $10^3 \text{ km}$
  - Supernova neutrinos are no longer oscillating
  - Different mass eigenstates produced at same instant arrive at separate times, depending on individual speeds
  - SN1987A  $\nu$  could have arrived  $10^{-4} \text{ s}$  apart

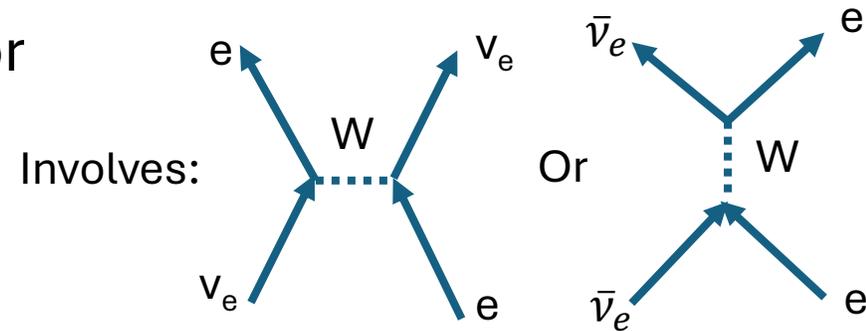
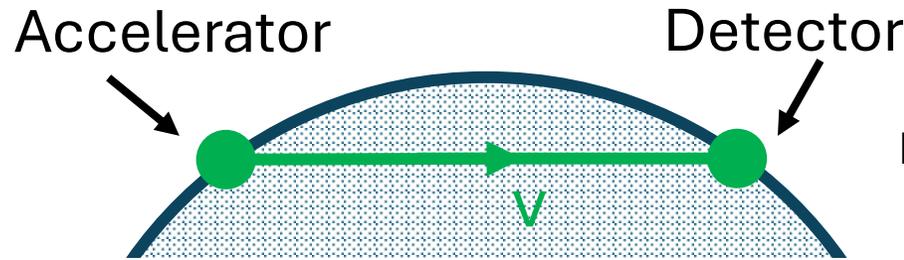
# Consider two mass model

- Two mass eigenstates  $\nu_1$  and  $\nu_2$ , with  $\Delta m_{21}^2 = \Delta m^2$
- Two flavor states ( $\nu_e$  and  $\nu_\mu$ )
- Mixing matrix is  $U = \begin{pmatrix} U_{e1} & U_{e2} \\ U_{\mu1} & U_{\mu2} \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$
- Recall  $P(\nu_\alpha \rightarrow \nu_\beta) = |\text{Amp}[\nu_\alpha \rightarrow \nu_\beta]|^2$ 

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2 \left( \Delta m_{ij}^2 \frac{L}{4E} \right)$$

$$+ 2 \sum_{i>j} \text{Im}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin \left( \Delta m_{ij}^2 \frac{L}{2E} \right)$$
- Therefore  $P(\nu_\mu \rightarrow \nu_e) = 4 \sin \theta \cos \theta \cos \theta \sin \theta = \sin^2(2\theta) \sin^2 \left( \Delta m_{12}^2 \frac{L}{4E} \right)$ .
- As before in vacuum.... Now what if there is matter?

# Neutrino flavor change in matter



- Coherent forward scattering via W-exchange leads to an extra interaction potential for  $\nu_e$

$$V_W = \begin{cases} +\sqrt{2}G_F N_e & \text{for } \nu_e \\ -\sqrt{2}G_F N_e & \text{for } \bar{\nu}_e \end{cases}$$

$G_F$  = Fermi constant  
 $N_e$  = electron density

- Raises effective mass of  $\nu_e$ , lowers for  $\bar{\nu}_e$

# Also get a neutral current contribution

- Z boson interaction for neutron, electron and proton leads to an extra interaction for all neutrino flavors
- Electron and proton contributions are equal and opposite, so cancel
- Contribution from interactions on neutrons

$$V_Z = +\sqrt{2}/2 G_F N_n \text{ for } \nu \text{ and } \bar{\nu}.$$

# Start from Schroedinger equation in lab frame

$$i \frac{\partial}{\partial t} |\nu(t)\rangle = \mathcal{H} |\nu(t)\rangle$$

- Where the Hamiltonian  $\mathcal{H}$  is a 2x2 matrix for our 2-neutrino model
- The vacuum component  $\mathcal{H}_{vac}$  is

$$\langle \nu_\mu | \mathcal{H}_{vac} | \nu_e \rangle = \sum_i \langle U_{\mu i} \nu_i | \mathcal{H}_{vac} | U_{ei} \nu_i \rangle = \sum_i U_{\mu i}^* U_{ei} E_i \langle \nu_i | \nu_i \rangle$$

$$\langle \nu_\mu | \mathcal{H}_{vac} | \nu_e \rangle = \sum_i U_{\mu i}^* U_{ei} \sqrt{p^2 + m_i^2}$$

- We can evaluate each of the four terms in  $\mathcal{H}_{vac}$  in relativistic approximation where  $\sqrt{p^2 + m_i^2} = \left( p + \frac{m_i^2}{2p} \right)$

# Vacuum Hamiltonian

- After a page of algebra you can find

$$\mathcal{H}_{vac} = \frac{\Delta m^2}{4p} \begin{bmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} + \left( p + \frac{m_1^2 + m_2^2}{4p} \right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Free to subtract a multiple of identity matrix, and for relativistic neutrinos  $p=E$ , thus

$$\mathcal{H}_{vac} = \frac{\Delta m^2}{4E} \begin{bmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$$

- Now in matter we have the Hamiltonian

$$\mathcal{H}_M = \mathcal{H}_{vac} + V_W \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + V_Z \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- As before subtract multiples of identity matrix, and write as

$$\mathcal{H}_M = \mathcal{H}_{vac} + V_W \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

# Hamiltonian in matter

- Plugging in  $V_W$  and  $\mathcal{H}_{\text{vac}}$  we find

$$\mathcal{H}_M = \frac{\Delta m^2}{4E} \begin{bmatrix} - \left( \cos 2\theta - \frac{V_W/2}{\Delta m^2/(4E)} \right) & \sin 2\theta \\ \sin 2\theta & \left( \cos 2\theta - \frac{V_W/2}{\Delta m^2/(4E)} \right) \end{bmatrix}$$

- Let  $x = \frac{V_W/2}{\Delta m^2/(4E)} = \frac{2\sqrt{2}G_F N_e E}{\Delta m^2}$ , pick  $X$  such that

$$\cos 2\theta_M = (\cos 2\theta - x)X$$

$$\sin 2\theta_M = (\sin 2\theta)X$$

$$\Delta m_M^2 = \frac{\Delta m^2}{X}$$

# Hamiltonian with matter effect

- We find

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - x)^2}$$

$$\Delta m_M^2 = \Delta m^2 \sqrt{\sin^2 2\theta + (\cos 2\theta - x)^2}$$

- Then

$$\mathcal{H}_M = \frac{\Delta m_M^2}{4E} \begin{bmatrix} -\cos 2\theta_M & \sin 2\theta_M \\ \sin 2\theta_M & \cos 2\theta_M \end{bmatrix}.$$

- And the oscillation probability becomes

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta_M \sin^2 \left( \frac{1.27 \Delta m_M^2 L}{E} \right)$$

# Neutrino flavor change in matter

- Fractional importance  $x$  of matter effect relative to oscillation with  $\Delta m^2$  is

$$x = \frac{\pm \sqrt{2} G_F N_e E}{\Delta m^2}$$

Interaction energy /  
Vacuum energy

- Grows with  $E$
- Sensitive to sign of  $\Delta m^2$
- Reverses for anti-neutrino
  - Last effect is a “fake CP violation” which must be accounted for

# The Borexino detector @ LNGS

Laboratori Nazionali del Gran Sasso

Active volume:  
280 tons of liquid scintillator.

Detection principle



Elastic scattering off the electrons of the scintillator.  
Threshold at  $\sim 60$  keV  
(electron energy)

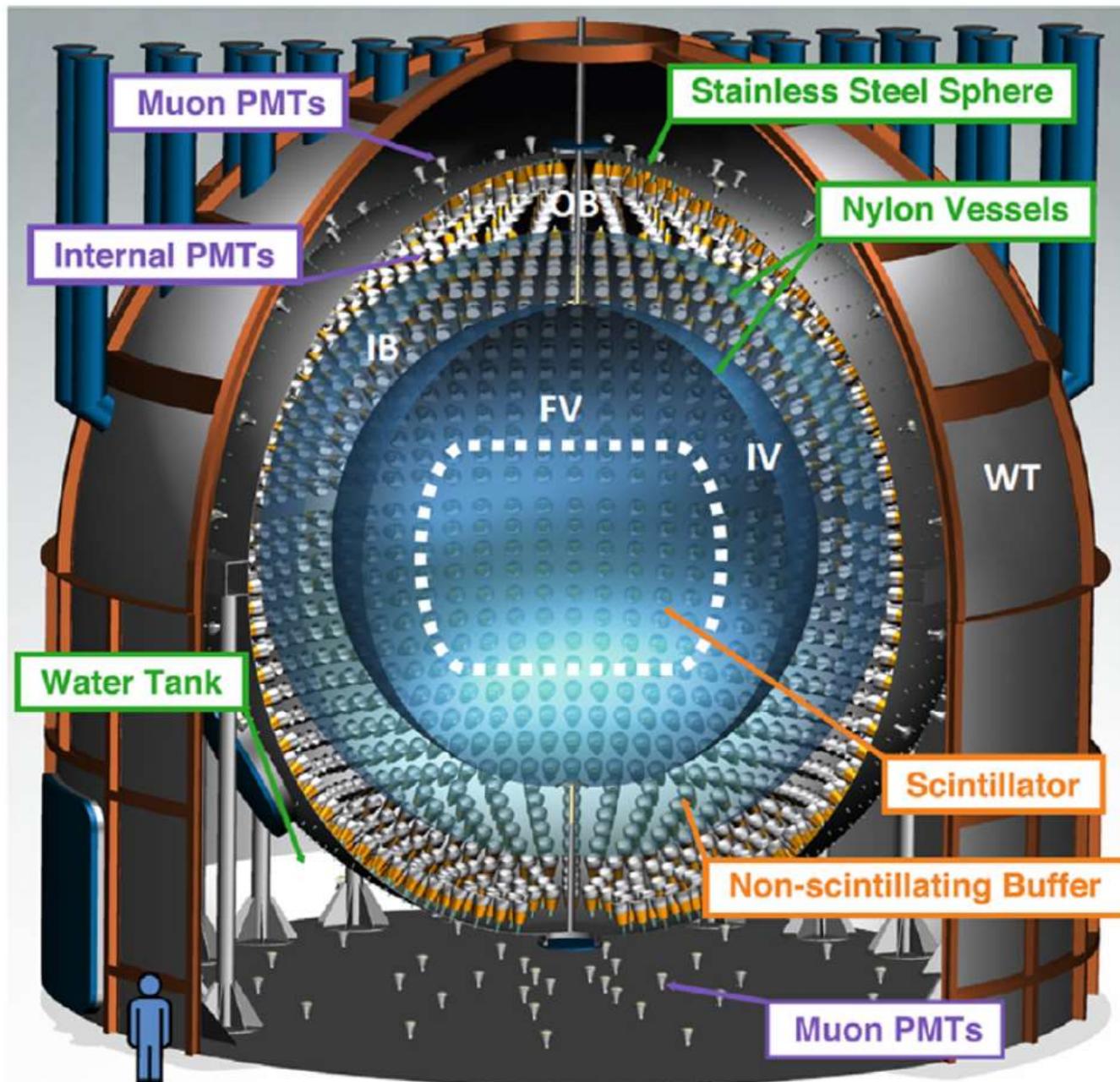
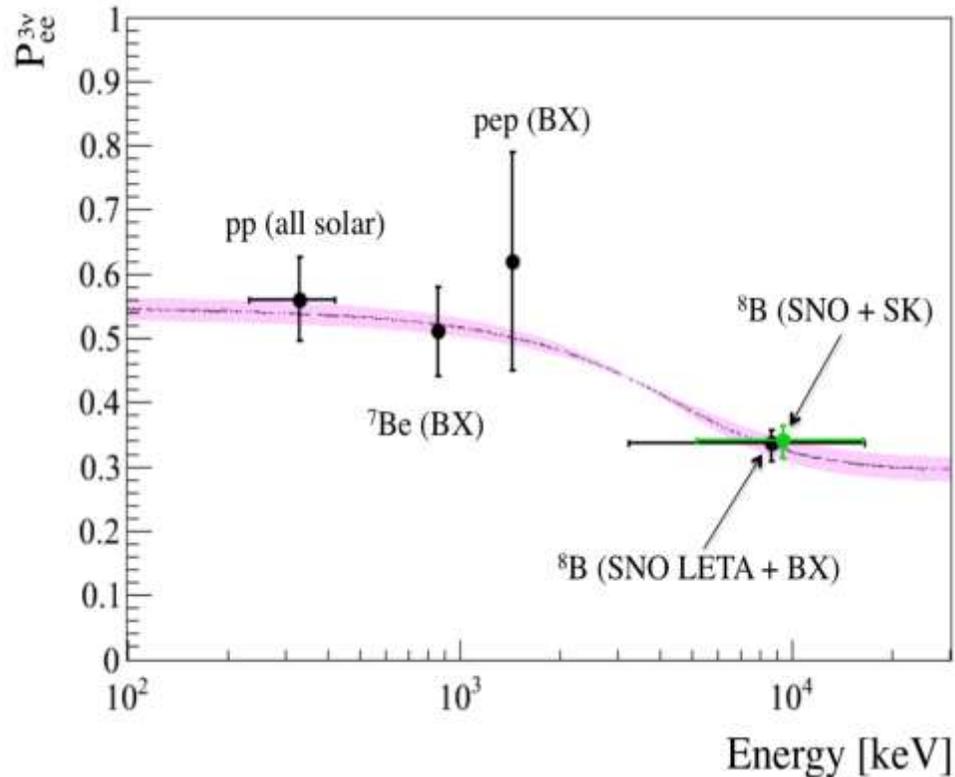


Figure from Borexino collaboration

# The significance of $P(\nu_e \rightarrow \nu_e)$

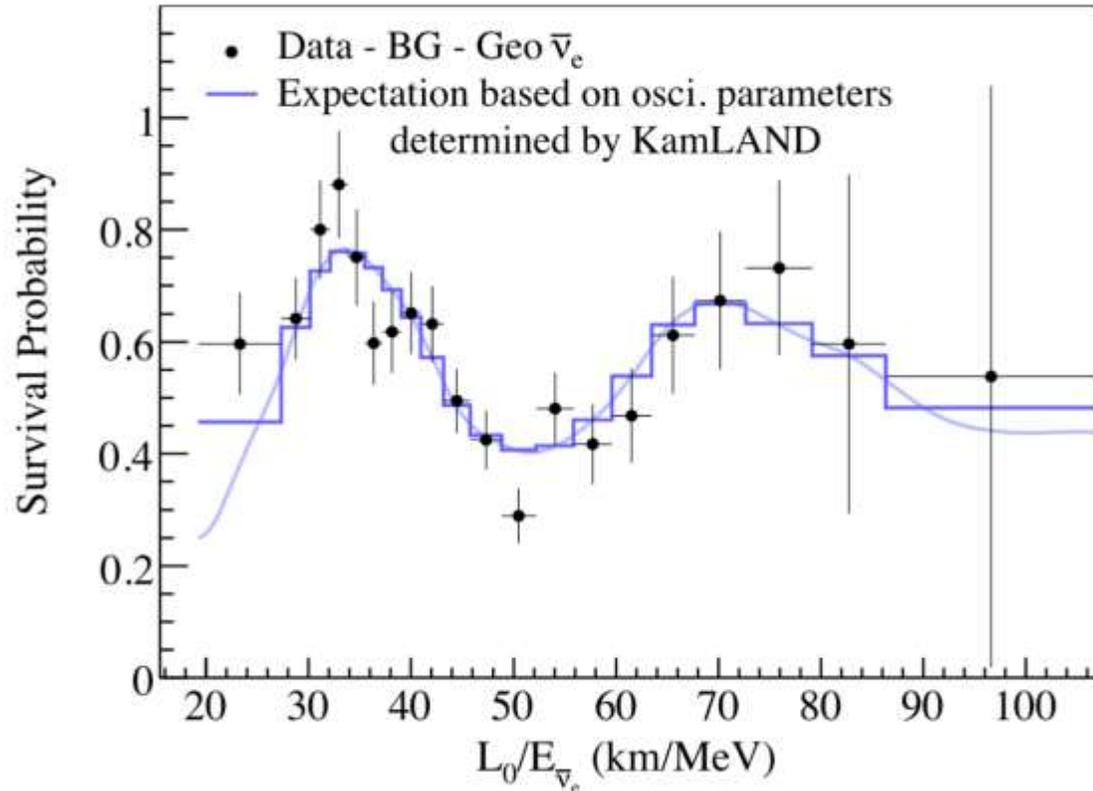


Phys. Rev. D 89:11 (2014) 11207.

- For SNO-energy range solar  $\nu$ , there is a strong solar matter effect
- A solar  $\nu$  is born in the core of the sun as a  $\nu_e$
- Emerging from sun, there is a 91% probability it is a  $\nu_2$
- Then  $P(\nu_e \rightarrow \nu_e)$  at earth is:  
 $|\langle \nu_e | \nu_e \rangle|^2 = |U_{e2}|^2 \cong 0.3$
- Solar  $\nu_e$  survival to lower energy measured by Borexino

# Kamland detector

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2(\theta_{\text{sol}}) \sin^2 \left[ 1.27 \Delta m_{\text{sol}}^2 \frac{L}{E} \right]$$



Phys. Rev. Lett 100 (2008) 221803.

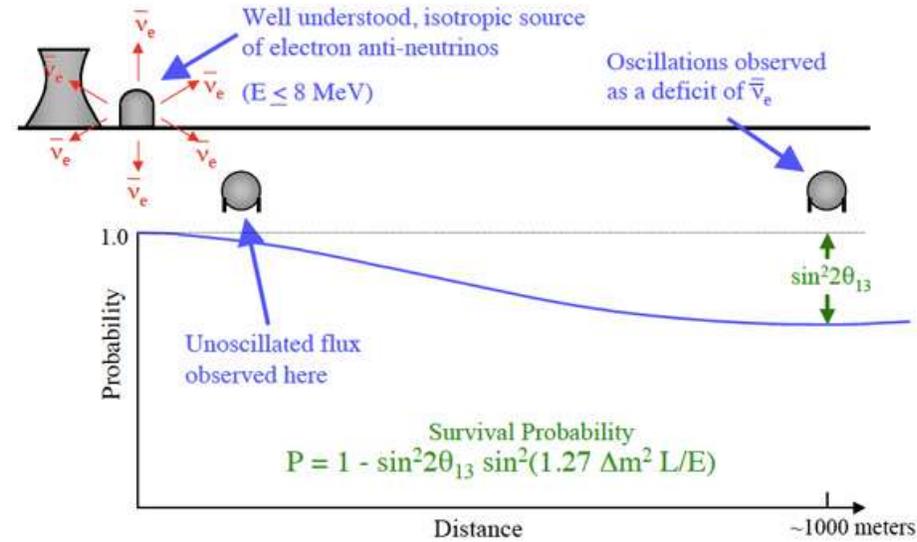


- Studied anti-neutrinos produced by Japanese nuclear reactors ~180 km away
- $\chi_{\text{matter}} < 10^{-2}$
- Survival probability oscillates as  $L/E$  as expected

# The reactor sector

km-baseline reactor experiments:

- ◆ more powerful reactors
- ◆ larger detector volume
- ◆ 2-8 detectors at 100 m – 1 km



2 cores + 1 ND + 1 FD



6 cores + 4 ND + 4FD



6 cores + 1 ND + 1 FD

# Reactor neutrino experiments

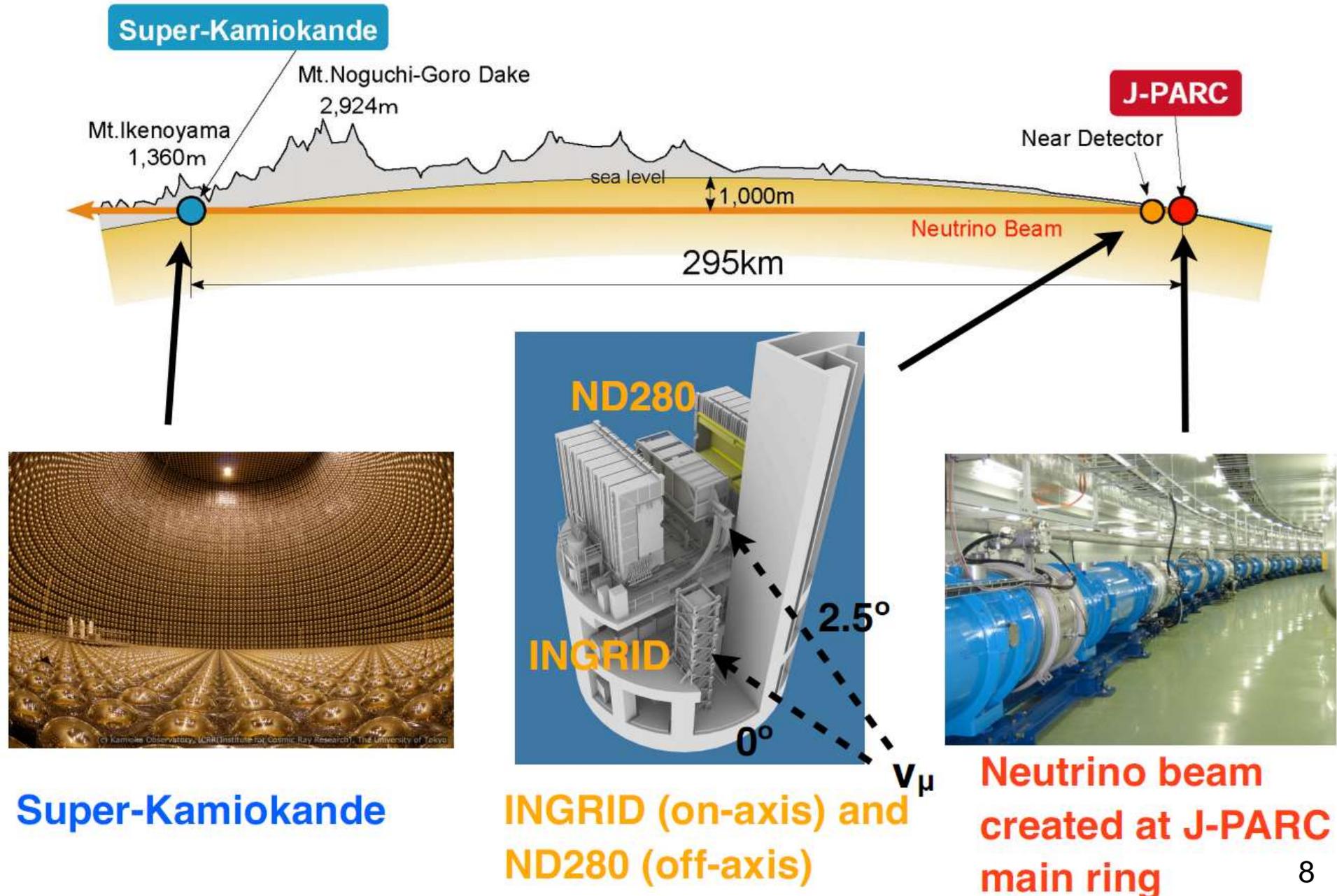
- Reactor  $\nu_e$  have  $E \sim 3$  MeV and  $L \sim 1.5$  km
  - $\sin^2 \left[ 1.27 \frac{1.5 \text{ km}}{3 \text{ MeV}} \Delta m^2 \right]$ :
    - Sensitive to  $1/400 \text{ eV}^2$  neutrinos (atmospheric)
    - Not sensitive to  $1/13000 \text{ eV}^2$  (solar neutrinos)

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - 4|U_{e3}|^2(1 - |U_{e3}|^2)\sin^2 \left[ 1.27 \Delta m_{atm}^2 \frac{L}{E} \right]$$

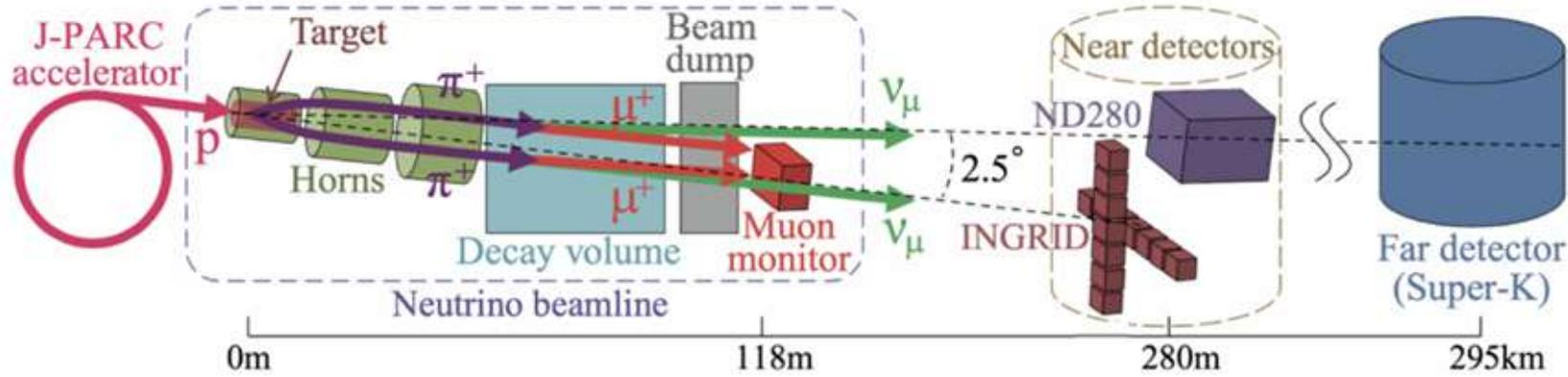
- Measurements find  $|U_{e3}|^2 \sim 0.02$

experiment	Location	$\sin^2 2\theta_{13}$
Daya Bay	China	$0.0853 \pm 0.0024$
RENO	South Korea	$0.0892 \pm 0.0063$
Double Chooz	France	$0.111 \pm 0.018$

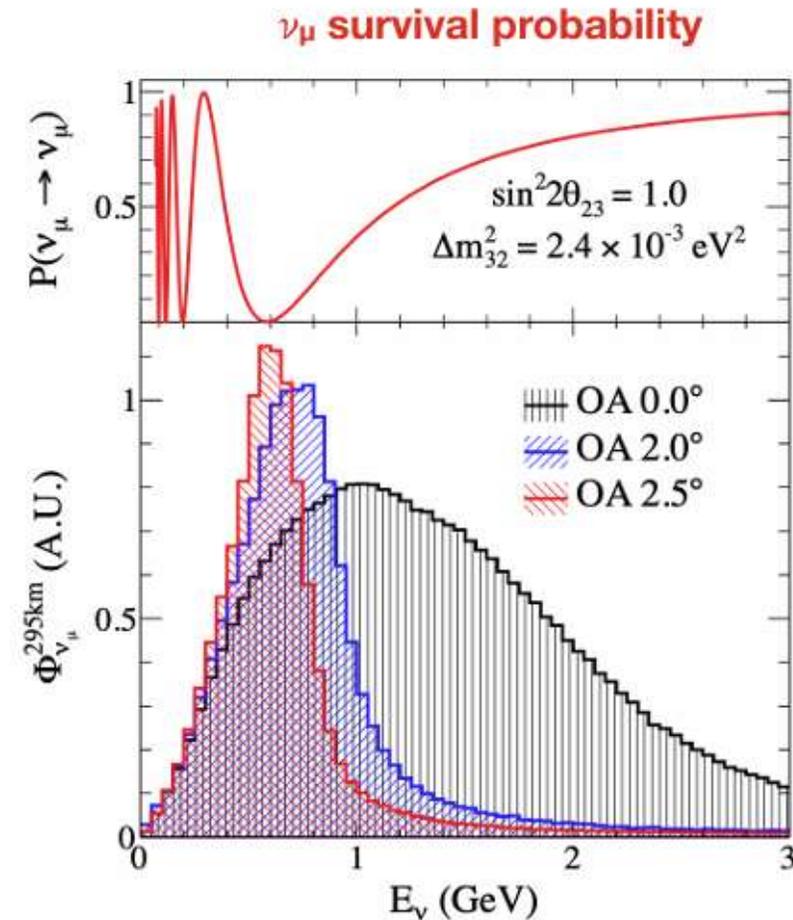
# T2K Experiment – Long baseline neutrino oscillations



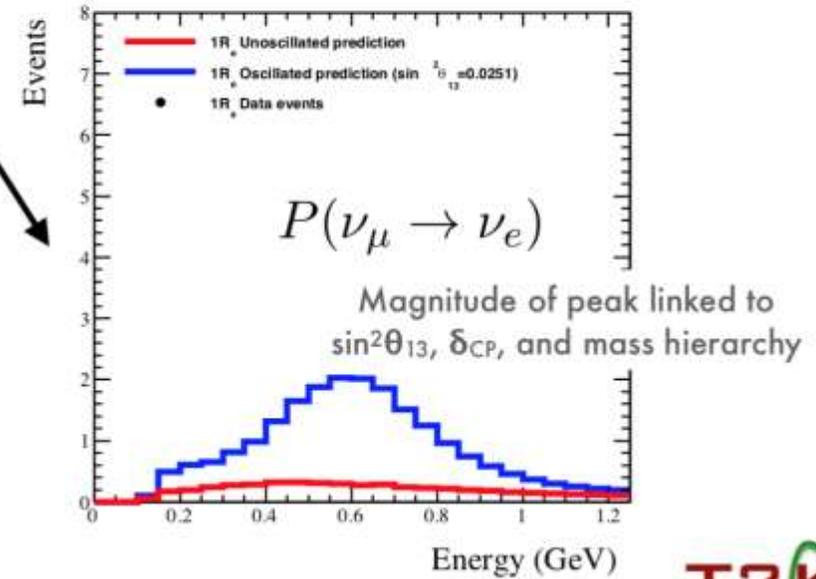
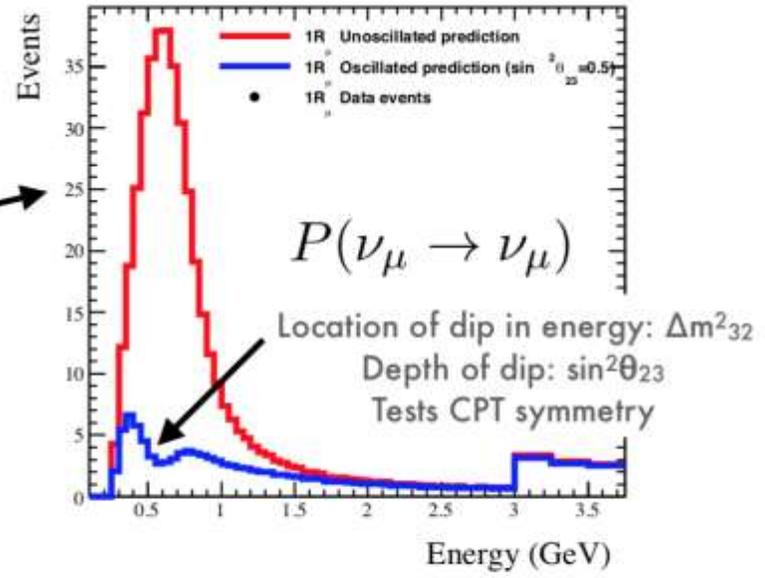
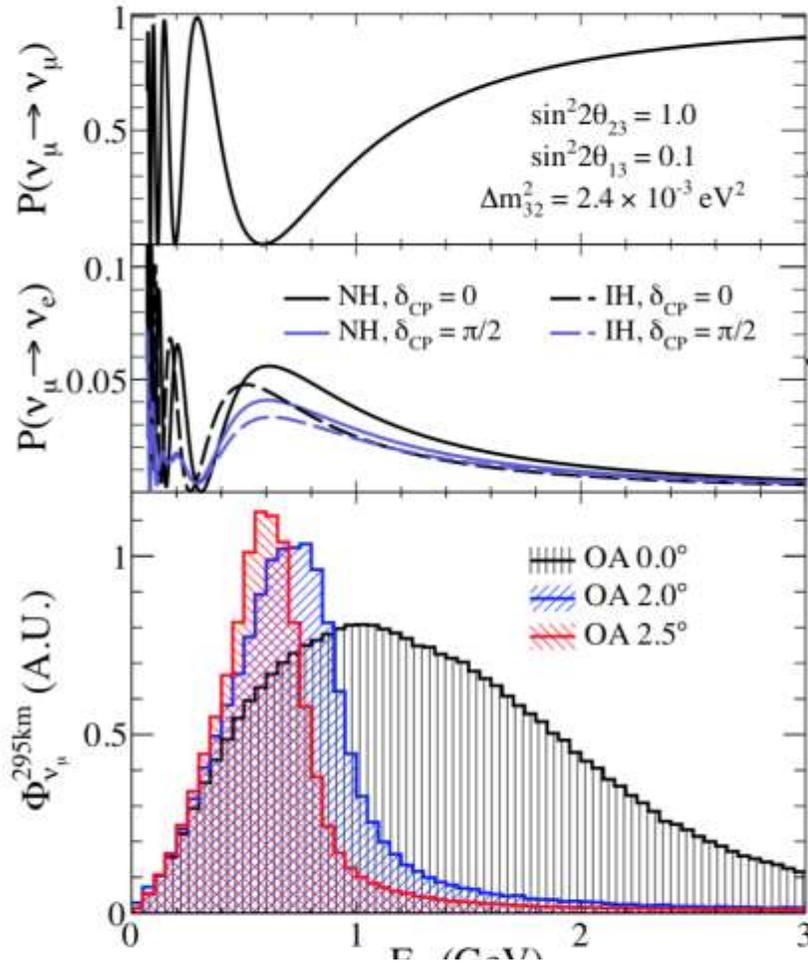
# T2K beamline



- 30 GeV proton beam from J-PARC Main Ring extracted onto a graphite target
- p+C interactions producing hadrons (mainly pions and kaons)
- Hadrons are focused and selected in charge by 3 electromagnetic horns
  - If  $\pi^+$  are focused  $\nu_\mu$  are produced by  $\pi^+ \rightarrow \mu^+ + \nu_\mu$
  - Changing the horn current we can produce  $\bar{\nu}_\mu$  from  $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$
- Off-axis technique  $\rightarrow$  detectors intercept a narrow-band beam at the maximum of the oscillation probability



# Long baseline neutrino experiments



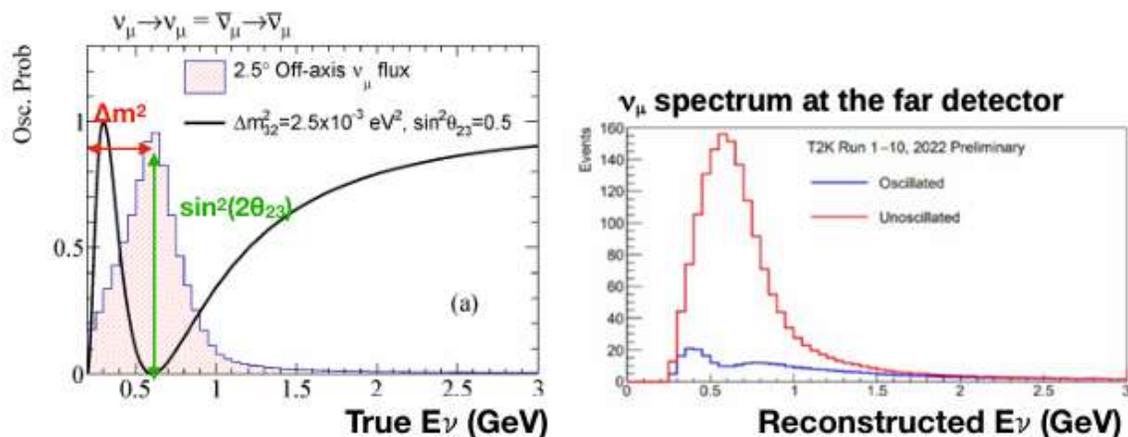
# Long Baseline Neutrino Oscillation Physics

## $\nu_\mu$ and $\bar{\nu}_\mu$ disappearance

$$P(\nu_\mu \rightarrow \nu_\mu) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu) = 1 - \sin^2(2\theta_{23}) \sin^2\left(1.27 \frac{\Delta m^2 L}{E}\right)$$

Same oscillation probability for  $\nu$  and  $\bar{\nu}$

Sensitive to  $|\Delta m^2_{32}|$  and to  $\sin^2(2\theta_{23}) \rightarrow$   
no sensitivity to mass ordering and  $\delta_{CP}$



## $\nu_e$ and $\bar{\nu}_e$ appearance

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \simeq \sin^2 \theta_{23} \frac{\sin^2 2\theta_{13}}{(A-1)^2} \sin^2[(A-1)\Delta_{31}]$$

$$\alpha = \Delta m^2_{21} / \Delta m^2_{31} \sim 1/30$$

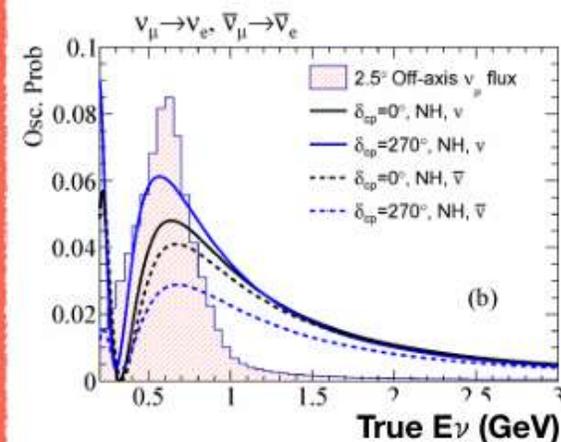
$$J_0 = \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \cos \theta_{13}$$

$$A = (\mp) 2\sqrt{2} G_F n_e E / \Delta m^2_{31}$$

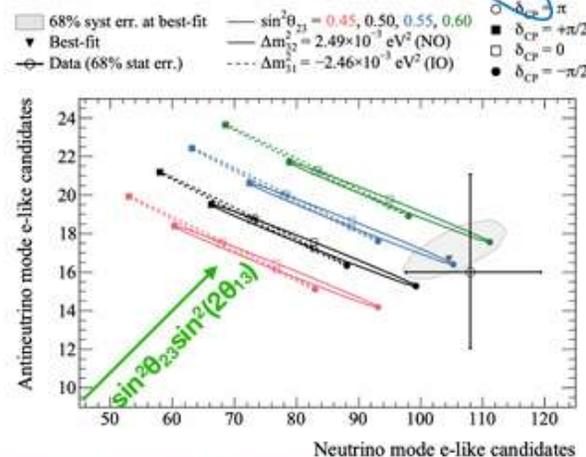
$$(\mp) \alpha \frac{J_0 \sin \delta_{CP}}{A(1-A)} \sin \Delta_{31} \sin(A\Delta_{31}) \sin[(1-A)\Delta_{31}]$$

$$+ \alpha \frac{J_0 \cos \delta_{CP}}{A(1-A)} \cos \Delta_{31} \sin(A\Delta_{31}) \sin[(1-A)\Delta_{31}] + O(\alpha^2)$$

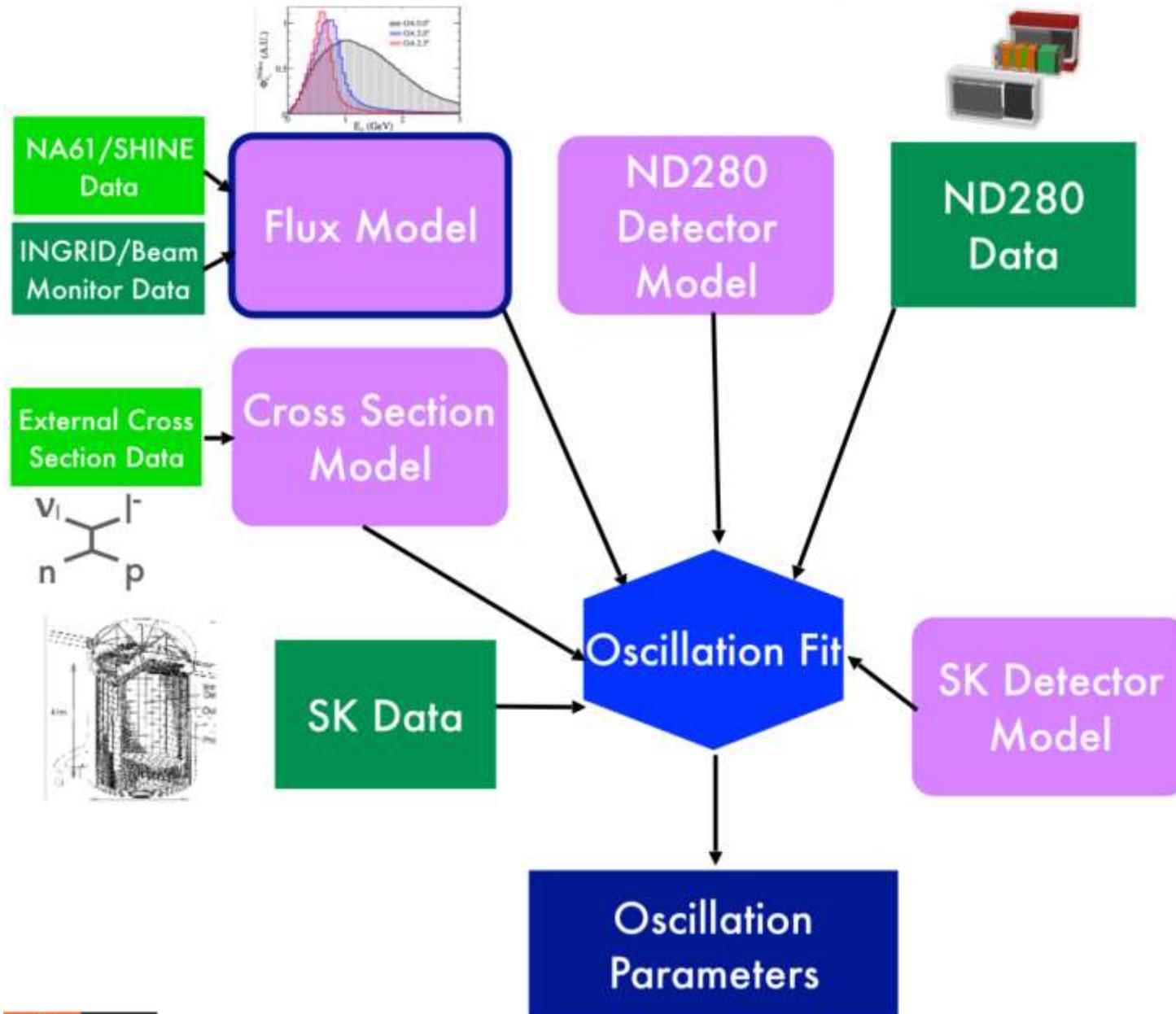
Sensitivity to  $\delta_{CP}$ , to the mass ordering and to the octant of  $\theta_{23}$



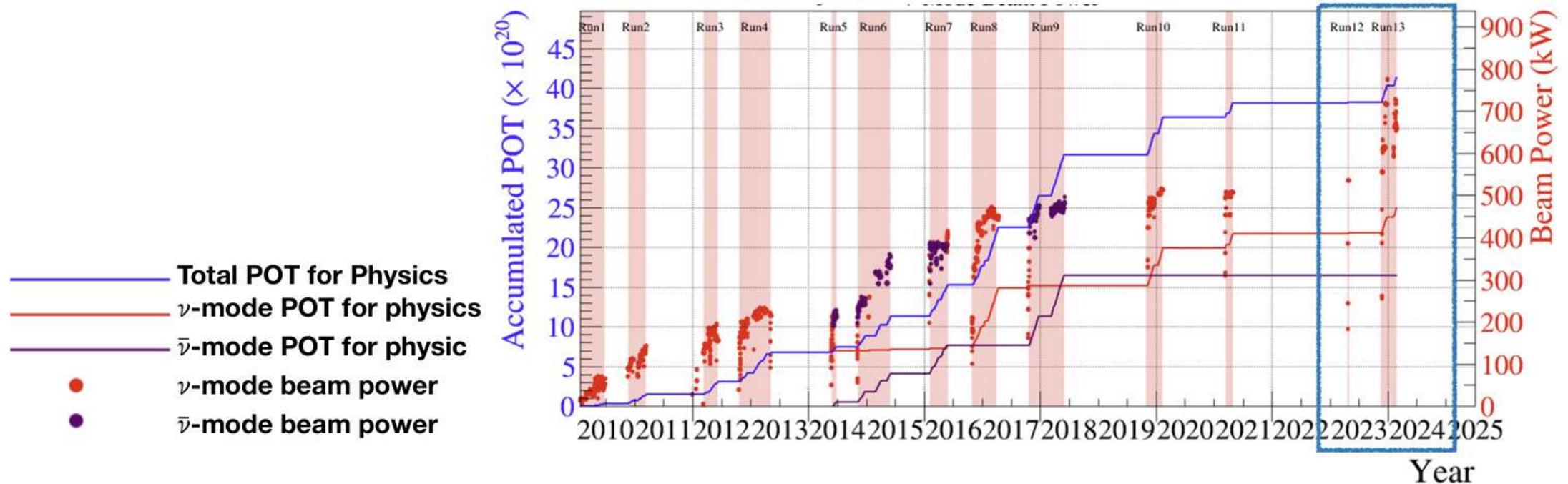
— Normal ordering  
... Inverted ordering



# T2K Oscillation Analysis



# T2K Data collected

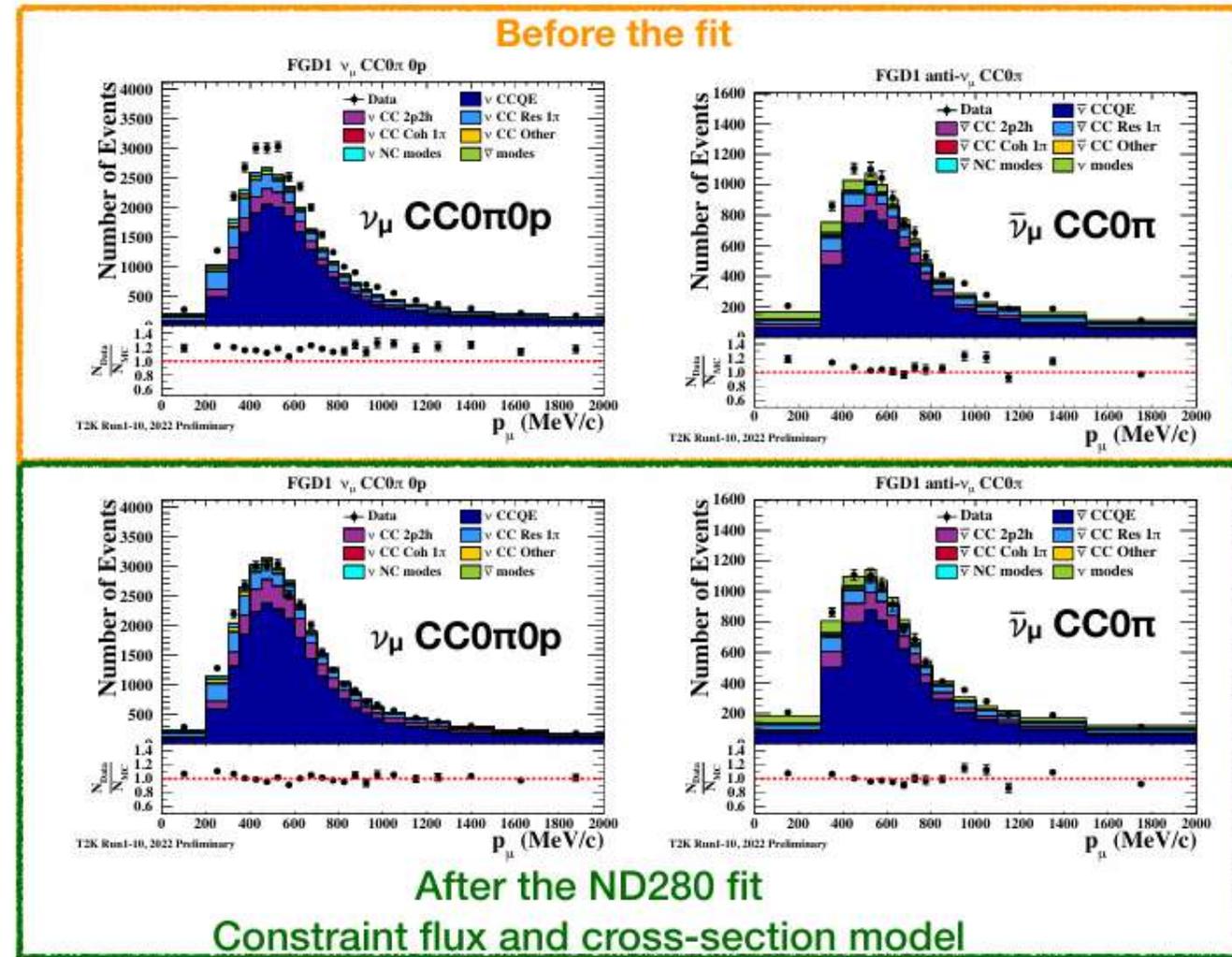


- Run 13 in December/February 2024, currently taking beam data
- Upgrades on the beamline → 750 kW reached in December 2023
- ND280 upgrade installed

# ND280 Fit

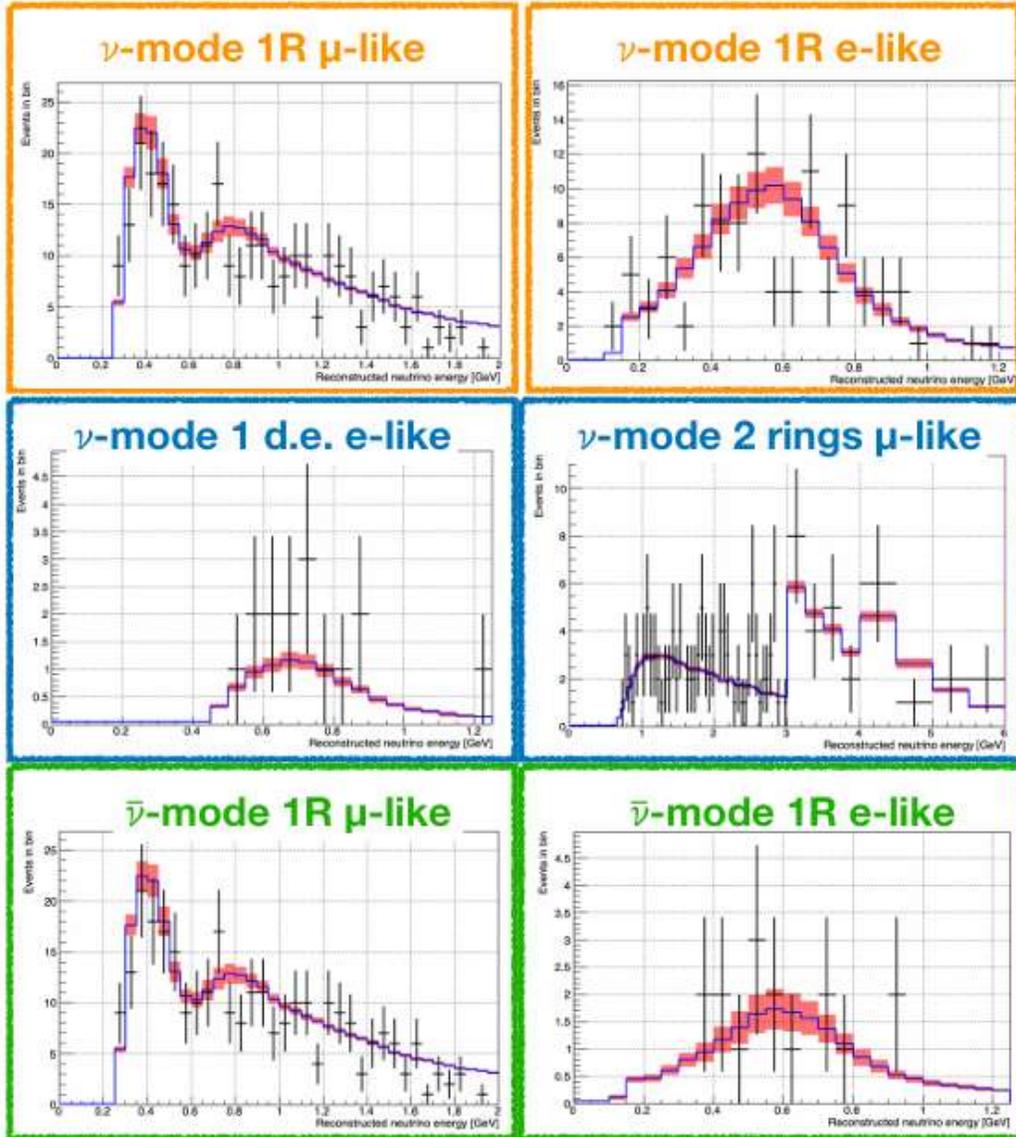


- ND280 magnetized detector
- Select interactions on CH (FGD1) and CH/Water (FGD2)
- Precise measurement of  $P_\mu$  and  $\theta_\mu$  with the TPCs
- Distinguish  $\nu$  from  $\bar{\nu}$  interactions thanks to the reconstruction of the charge of the lepton
- Separate samples based on number of reconstructed pions (CC0 $\pi$ , CC1 $\pi$ , CCN $\pi$ ), protons, photons, etc  $\rightarrow$  22 samples in total are used in the fit

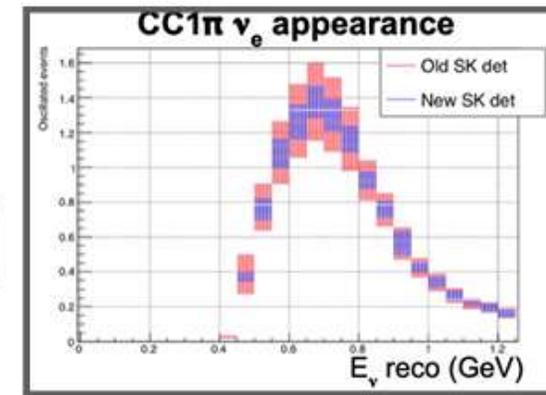


# Super-K accelerator neutrino detection

- 6 samples are selected at SK
  - 2 samples 1R  $\mu$ -like/e-like in  $\nu$ -mode  $\rightarrow$  CCQE enhanced
  - 2 samples CC1 $\pi$  enhanced (2 rings or with an additional decay electrons)
  - 2 samples 1R  $\mu$ -like/e-like in  $\bar{\nu}$ -mode  $\rightarrow$  CCQE enhanced
- New detector covariance matrix at SK  $\rightarrow$  significantly reduce systematics in the 1 Re+d.e. sample

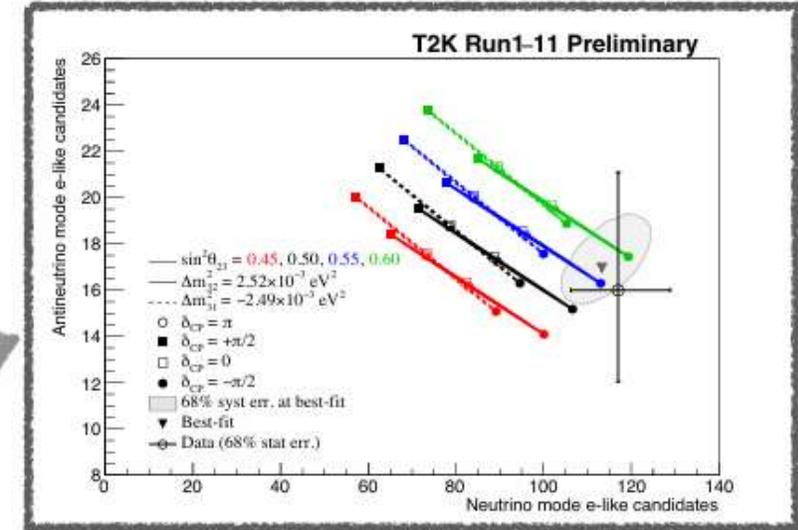


Sample	OA22	New results
$\nu$ -mode 1R $\mu$	3.4%	3.2%
$\nu$ -mode 1Re	5.2%	4.9%
$\nu$ -mode MR	4.9%	3.9%
$\nu$ -mode 1Re+d.e.	14.3%	6.3%
$\bar{\nu}$ -mode 1R $\mu$	3.9%	5.0%
$\bar{\nu}$ -mode 1Re	5.8%	6.7%

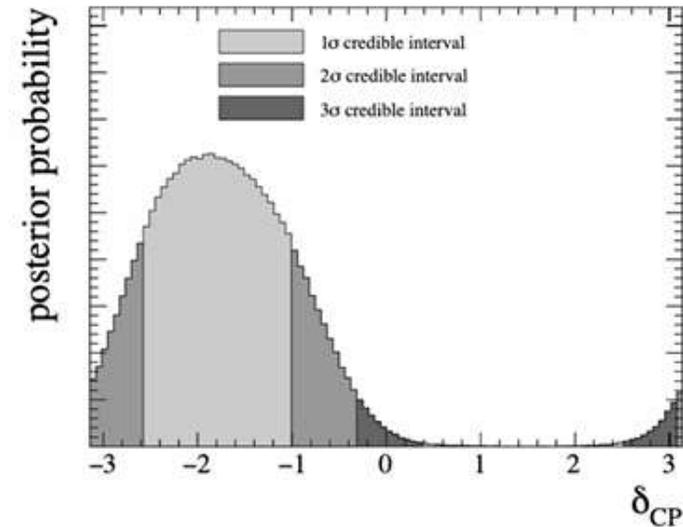
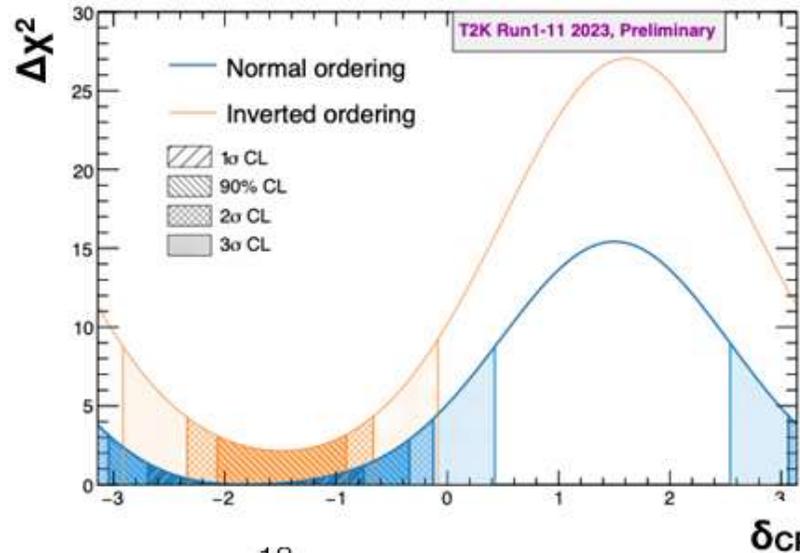


# Oscillation analysis results

Sample	$\delta_{CP}=-\pi/2$	$\delta_{CP}=0$	$\delta_{CP}=\pi/2$	$\delta_{CP}=\pi$	Data
$\nu$ -mode 1R $\mu$	417.2	416.3	417.1	418.2	357
$\nu$ -mode MR	123.9	123.3	123.9	124.4	140
$\bar{\nu}$ -mode 1R $\mu$	146.6	146.3	146.6	147.0	137
$\nu$ -mode 1Re	113.2	95.5	78.3	96.0	102
$\bar{\nu}$ -mode 1Re+d.e.	10.0	8.8	7.2	8.4	15
$\bar{\nu}$ -mode 1Re	17.6	20.0	22.2	19.7	16



Credible intervals marginalized over both hierarchies

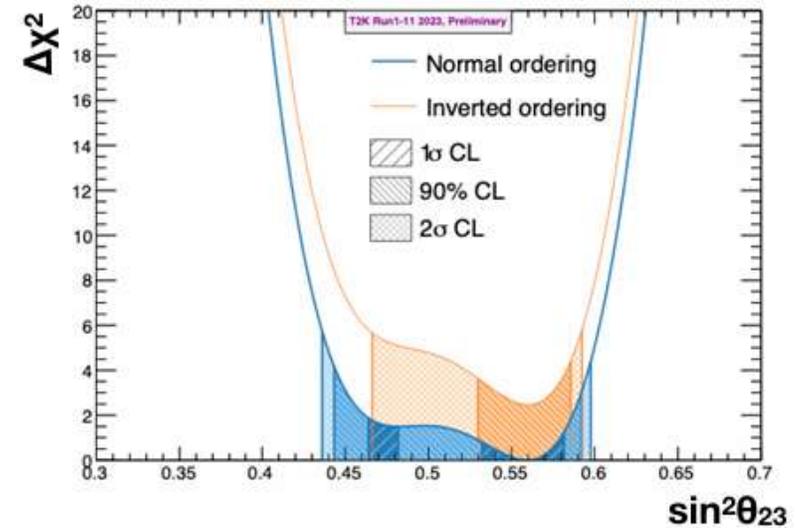


- Preference for  $\delta_{CP} \sim -\pi/2$  but CP conserving values are within the  $2\sigma$  interval

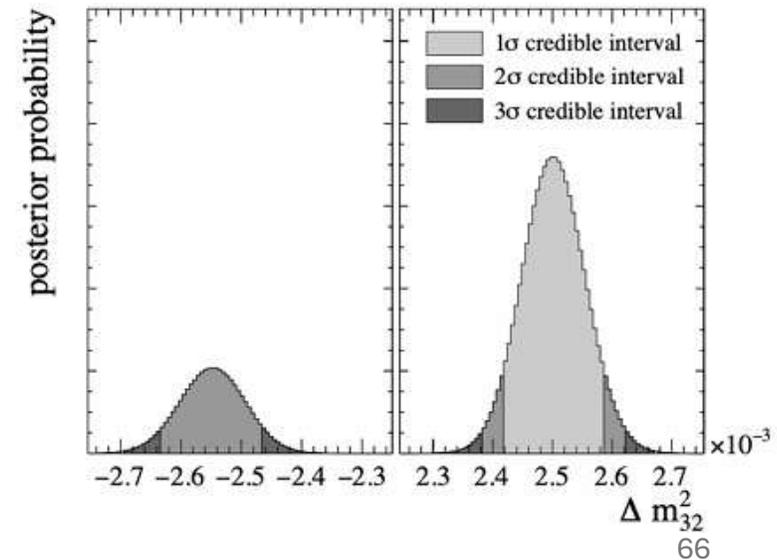
D. Carabadjac poster

# Mass ordering and $\theta_{23}$ octant

- Slight preference for normal ordering and upper octant but none of them is significant
  - Bayes factor NO/IO = 3.3
  - Bayes factor  $(\theta_{23} > 0.5) / (\theta_{23} < 0.5) = 2.6$

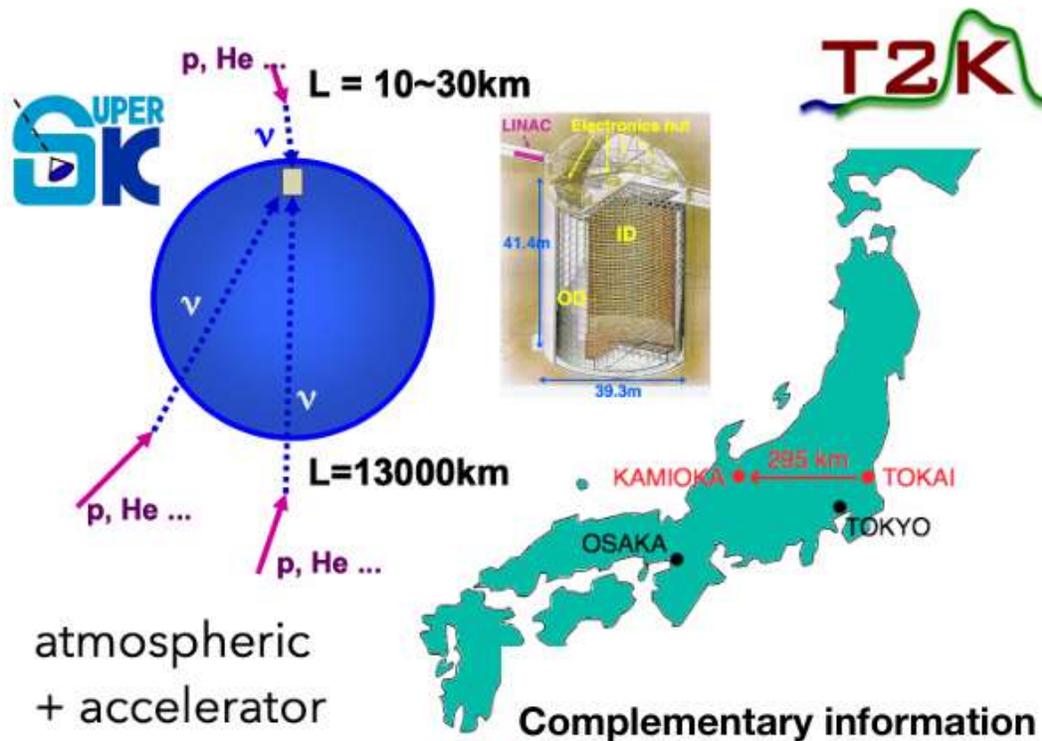


	$\sin^2 \theta_{23} < 0.5$	$\sin^2 \theta_{23} > 0.5$	Sum
NH ( $\Delta m_{32}^2 > 0$ )	0.23	0.54	0.77
IH ( $\Delta m_{32}^2 < 0$ )	0.05	0.18	0.23
Sum	0.28	0.72	1.00



# Joint analyses

- In 2023 we released two joint analyses
- T2K+NOvA combination → will be presented in the next talk
- T2K+SK combination

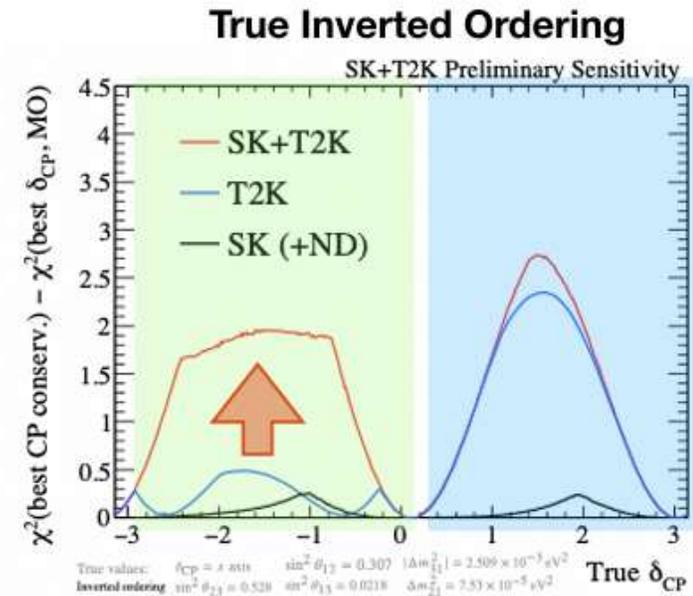
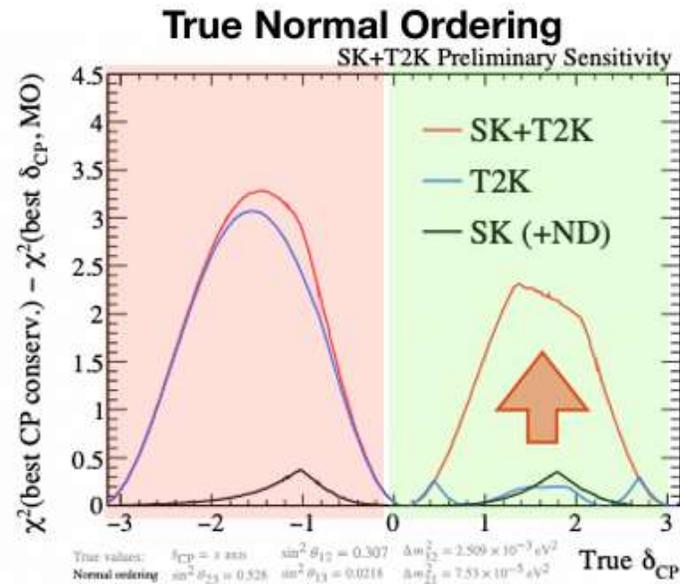
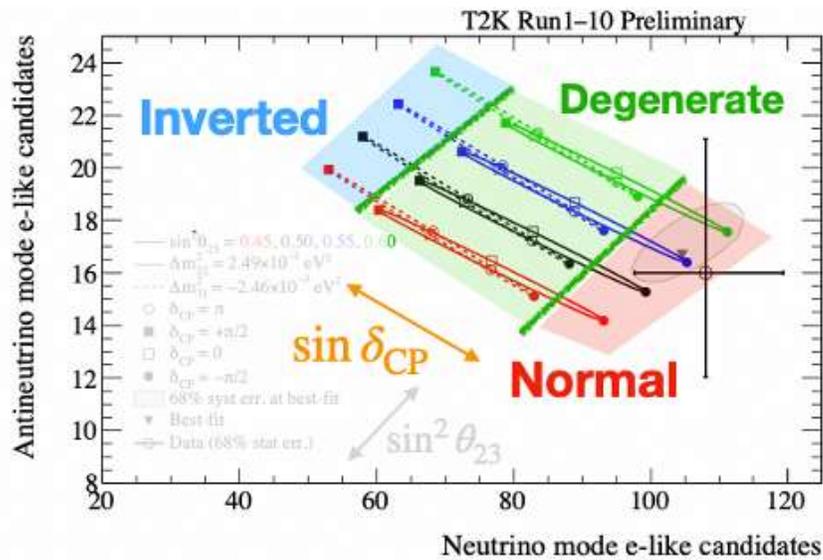


T2K data as in [Phys.Rev.D 108 \(2023\) 7, 072011](#) - (5 samples) POT:  $3.6 \times 10^{21}$

SK-IV data (18 samples) before Gd doping  
[PTEP 2019 \(2019\) 5, 053F01](#) - 3244 days (2008-2018)

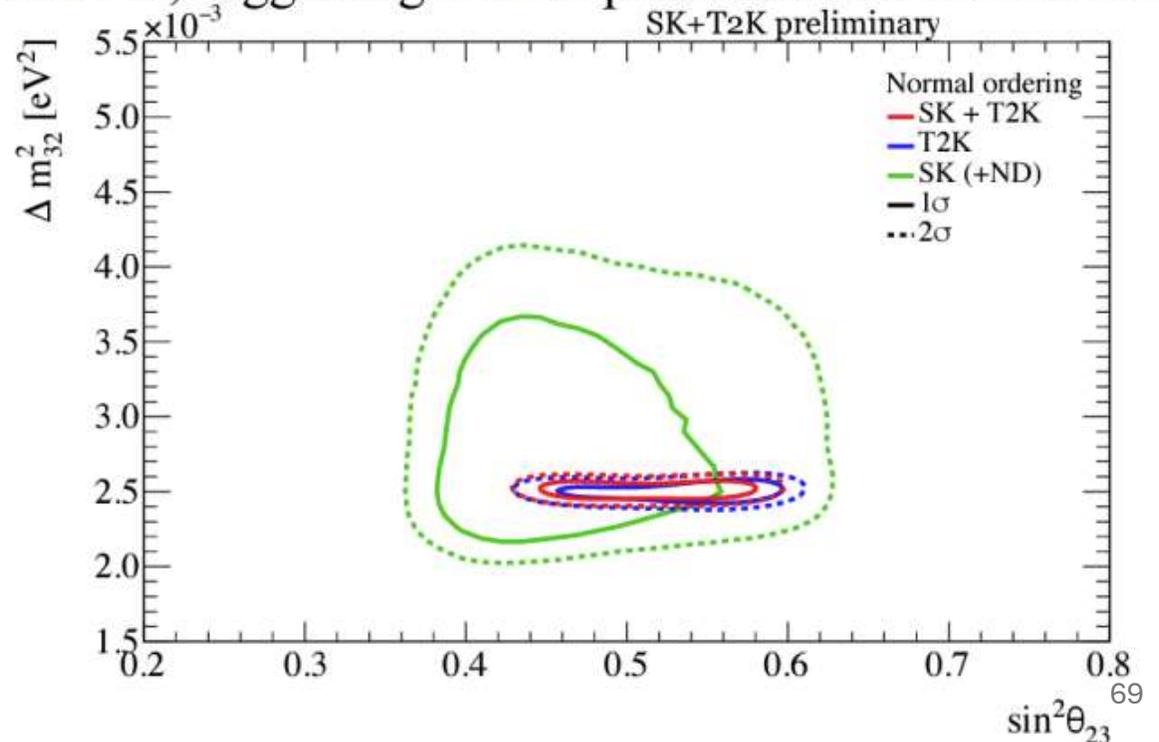
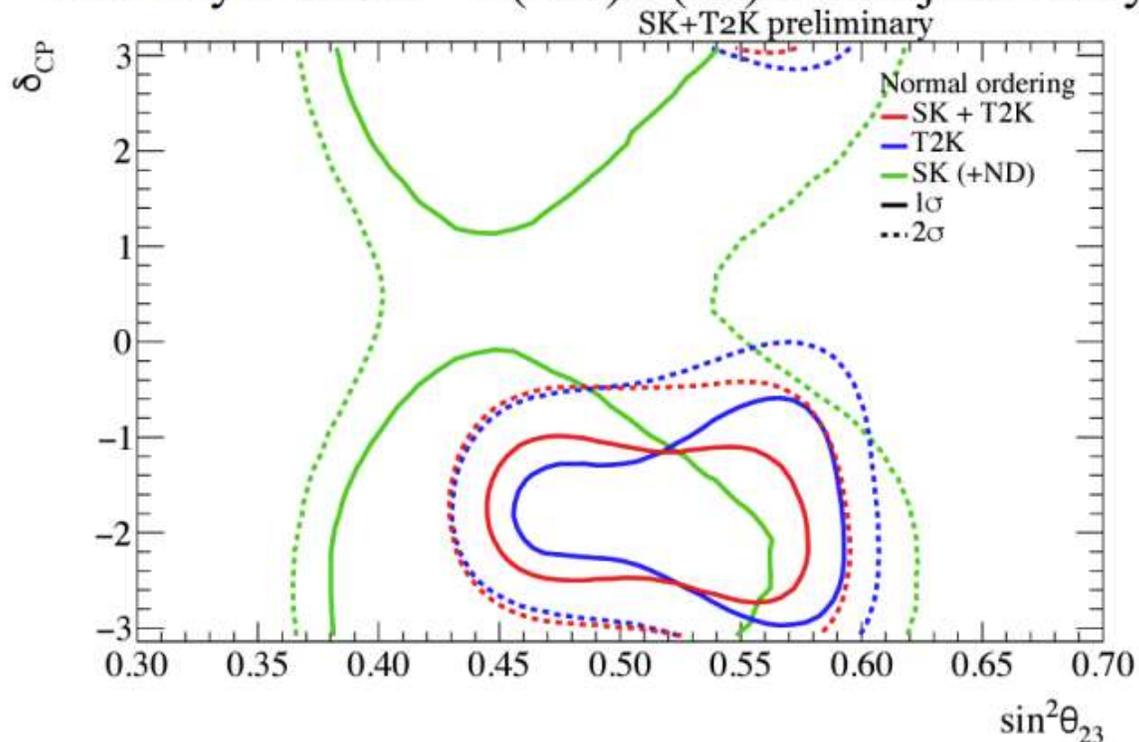
# T2K+SK joint analysis

- T2K has good sensitivity to  $\delta_{CP}$  but mild sensitivity to mass ordering
- SK has good constraint on mass ordering but not on  $\delta_{CP}$
- Adding SK atmospheric sample allows to break the degeneracies between the CP violation parameter  $\delta_{CP}$  and the mass ordering  $\rightarrow$  boost sensitivity to CP



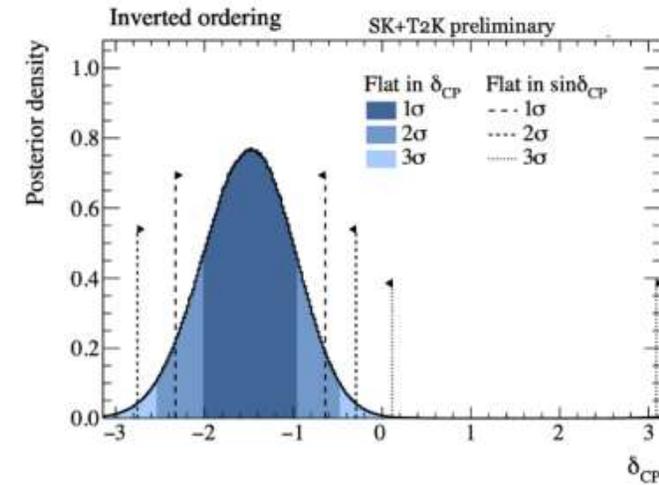
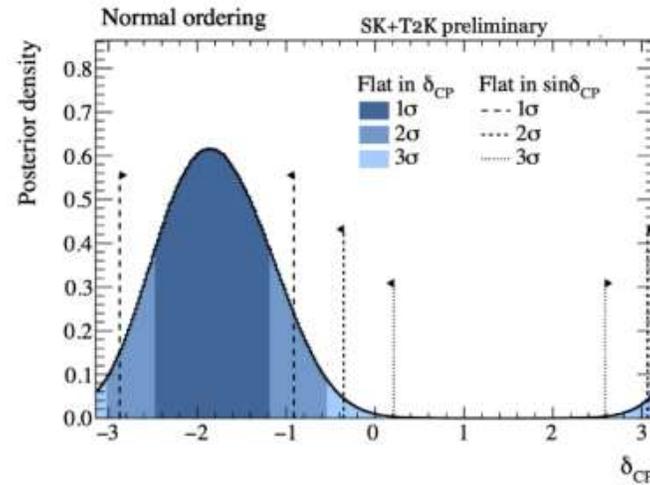
# Bayesian results

- The constraint of the joint SK-T2K data is mainly from the T2K samples. Combined with the Super-K samples, the constraint becomes stronger than in the individual T2K-only fit.
- T2K-only data fit shows a preference for the upper octant, while the Super-K-only data fit shows a preference for the lower octant. When the data from both experiments are combined, the results does not have a strong octant preference.
- MO Bayes factor =  $P(\text{NO})/P(\text{IO})$  in this joint analysis is  $\sim 9$ , suggesting a weak preference for normal MO



# $\delta_{CP}$ credible interval and Jarlskog invariant intervals

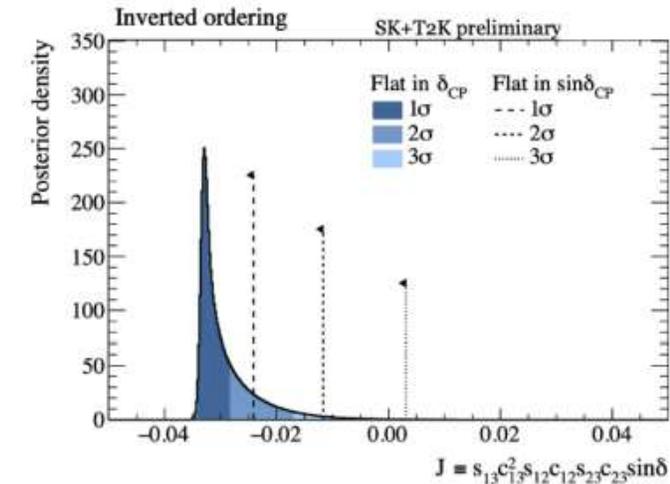
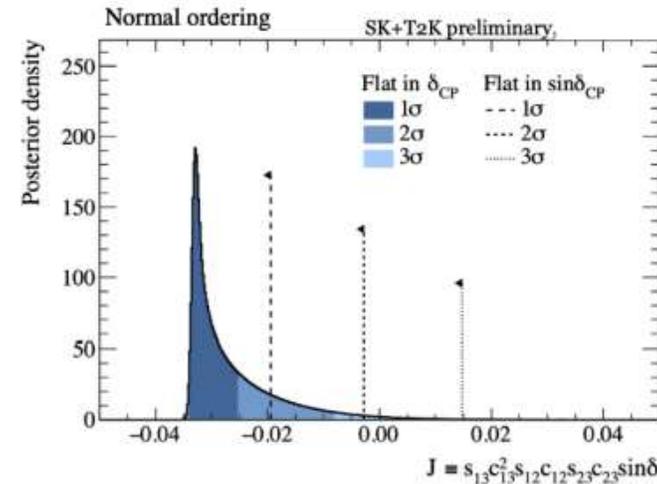
- $\delta_{CP} = 0$  or  $\pi$  is excluded by  $2\sigma$  with a flat  $\delta_{CP}$  prior.
- However,  $\delta_{CP} = \pi$  is not excluded at the  $2\sigma$  level in normal MO with a flat  $\sin \delta_{CP}$  prior.



## Jarlskog invariant credible intervals

$$J_{CP} = s_{13}c_{13}^2s_{12}c_{12}s_{23}c_{23} \sin \delta_{CP}$$

- $J_{CP} = 0$  is excluded at  $2\sigma$  with the flat  $\delta_{CP}$  prior.
- The exclusion of  $J_{CP} = 0$  at  $2\sigma$  is not robust with respect to possible biases seen in studies of alternative models for the flat prior of  $\sin \delta_{CP}$



# Conclusion of Lecture 1

- Reviewed history leading to discovery of neutrino oscillations
- Covered two and three neutrino oscillation theory
- Studied matter effect on neutrino
- Learned about current long-baseline neutrino experiment T2K