Dark Matter and Cosmology I: Introduction to Early Universe Energy Density Negotiations

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First, some discussion on dimensional analysis as a way to understand the essentials of anything.

$$\Delta X \Delta p \geq \frac{\hbar}{2}$$
$$\Delta E \Delta T \geq \frac{\hbar}{2}$$
$$\lambda_c = \frac{2\pi\hbar}{m}$$
$$\lambda_d = \frac{2\pi\hbar}{p}$$

Set $\hbar$=c=1

$$G = \frac{1}{M_{PL}^2} = \frac{8\pi}{m_{pl}^2} = 10^{-38}\ \mathrm{GeV}^{-2}$$
$$\mathrm{GeV} = \frac{1}{2 \times 10^{-14}\ \mathrm{cm}}$$
$$\mathrm{GeV} = \frac{1}{7 \times 10^{-25}\ \mathrm{s}}$$

This is also called natural units.

$$[E] \sim \left[\frac{1}{X}\right] \sim \left[\frac{1}{T}\right]$$
$$[X] \sim [T] \sim \left[\frac{1}{E}\right]$$

proton mass ~GeV

How much stuff is there in stuff?

Stuff in a nucleus or neutron star?

(Units of GeV^4)

Stuff around us (bonus: α_{em}, m_e, m_p).

(Units of GeV^4)

How much stuff is there in stuff?

Stuff in a nucleus or neutron star?

(Units of GeV^4)

application: mean free path of WIMP in NS?
(for finding dark matter)

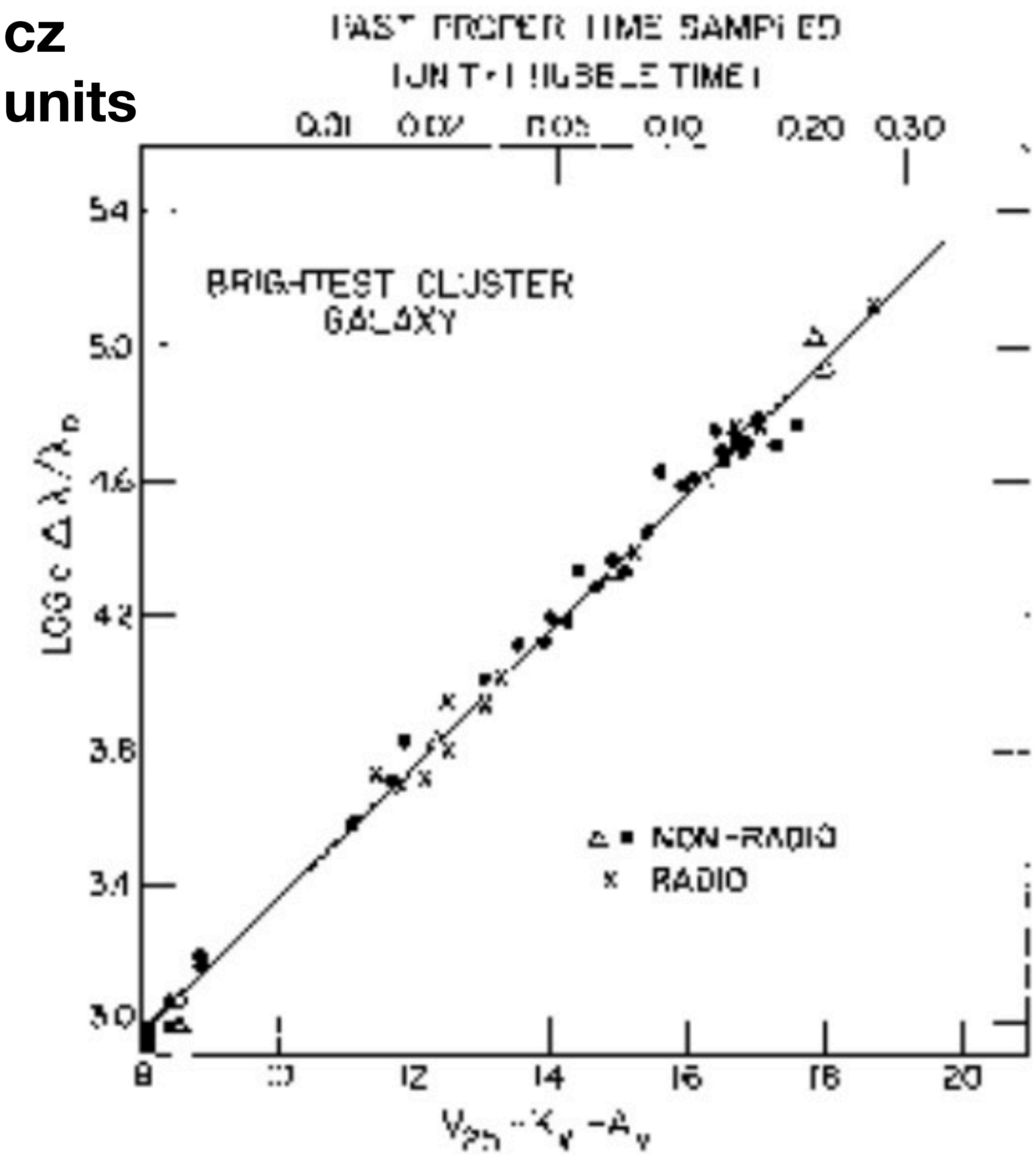
Stuff around us (bonus: α_{em}, m_e, m_p).

(Units of GeV^4)

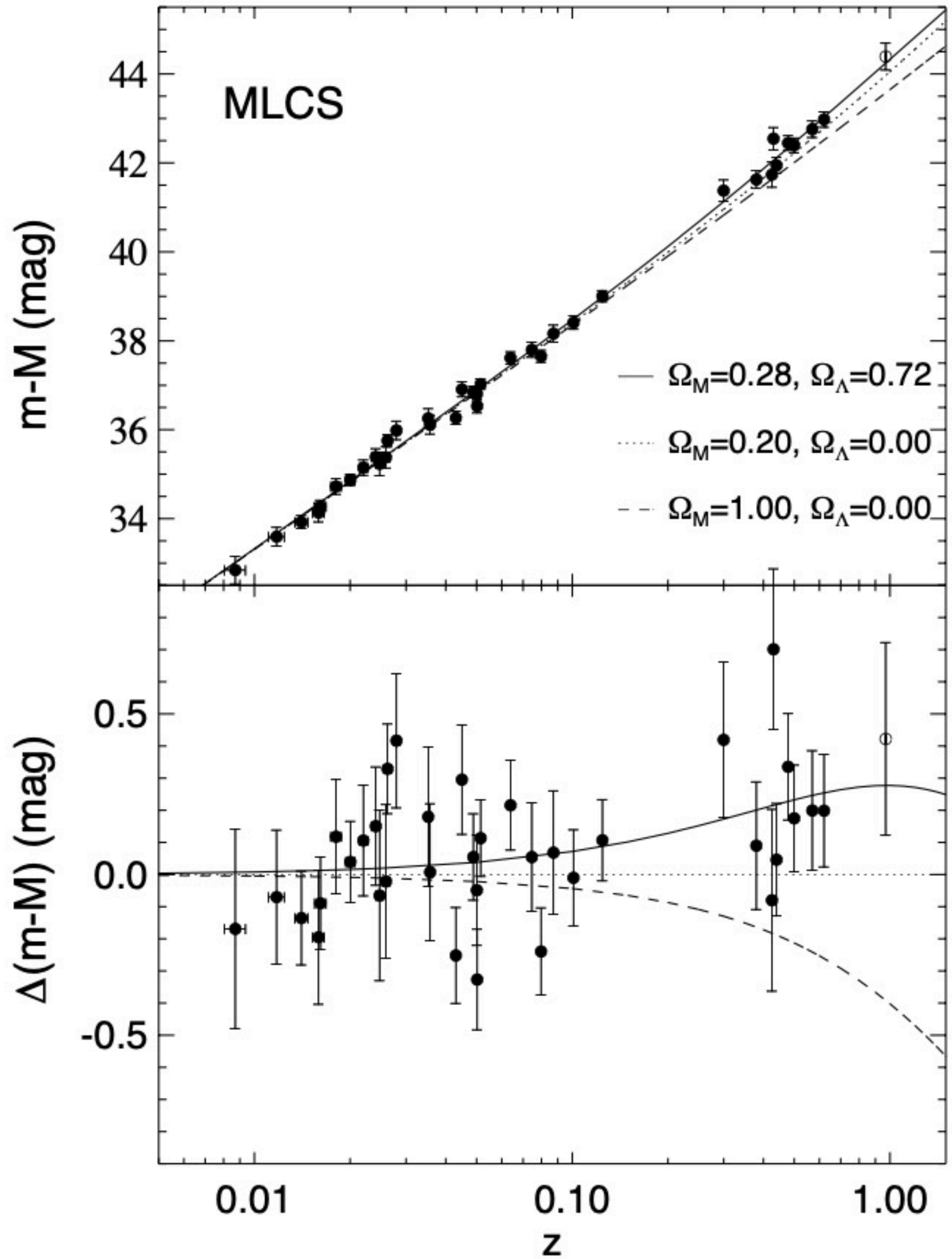
application: energy density of stuff
(for laptops and cars)

Our universe is rather isotropic and homogeneous

cz units

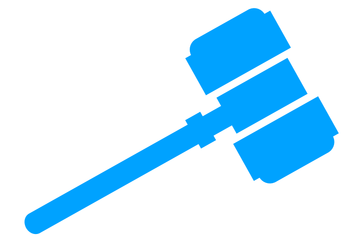


$$H \equiv \frac{\dot{a}}{a}$$

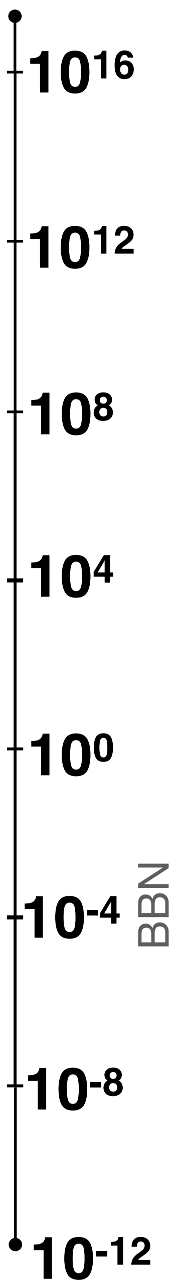


(Dodelson - Modern Cosmology)

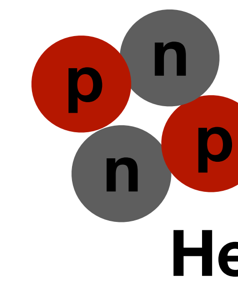
BBN and hints at isotropy



Assume universe started as a hot dense, place



Then at temperatures of around an MeV, neutrons and protons could begin to combine into deuterium and Helium. We have observed a baryon to photon ratio $\eta \equiv n_b/n_\gamma \sim 6 \times 10^{-10}$

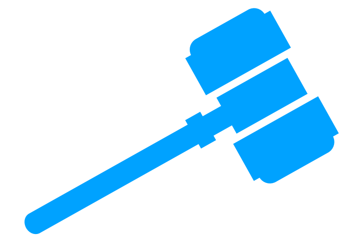


At T~0.8 MeV, weak forces (e.g. ν + p <—> n + e^+) drop out of equilibrium. In the absence of outside forces, the relative abundance of protons and neutrons at this temperature will depend on their mass splitting $\Delta m_{np} \sim 1.3$ MeV according to a Boltzmann distribution

$$n_n/n_p \sim e^{-\Delta m_{np}/T_w} \sim 1/6$$

From here, protons and neutrons would combine to form Deuterium and then Helium. However, if high energy photons are around, they would tend to split apart Deuterium which has binding energy $BE_D \sim 2.2$ MeV.

BBN and hints at isotropy (ii)



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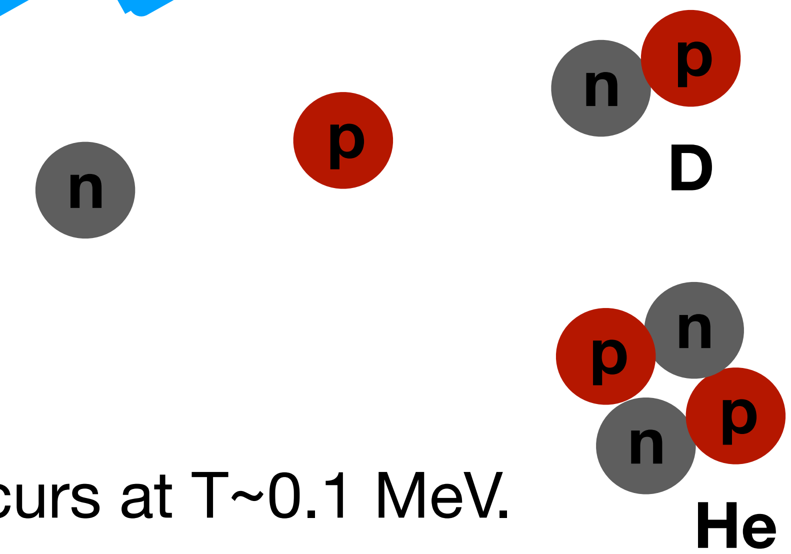
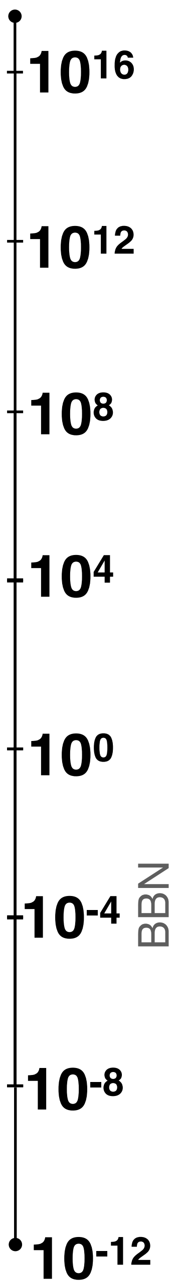


$$\eta \equiv n_b/n_\gamma \sim 6 \times 10^{-10}$$



Since photons outnumber baryons, the condition for deuterium to form is $\eta^{-1} e^{-BE_D/T} \lesssim 1$, which occurs at T~0.1 MeV.

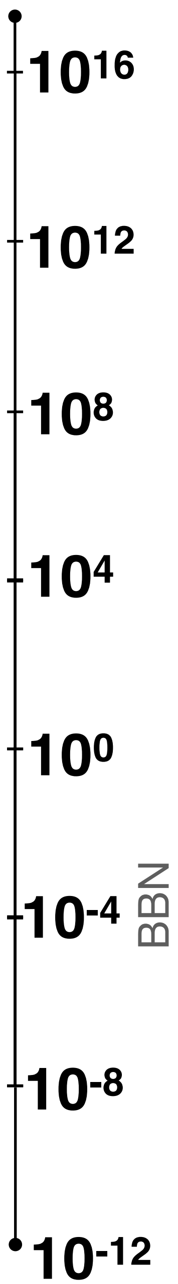
This temperature corresponds with a timescale of ~100 seconds. We can estimate that a ~10% fraction of the neutrons (with lifetime 15 minutes) have decayed by then, so that $n_n/n_p \sim 1/7$

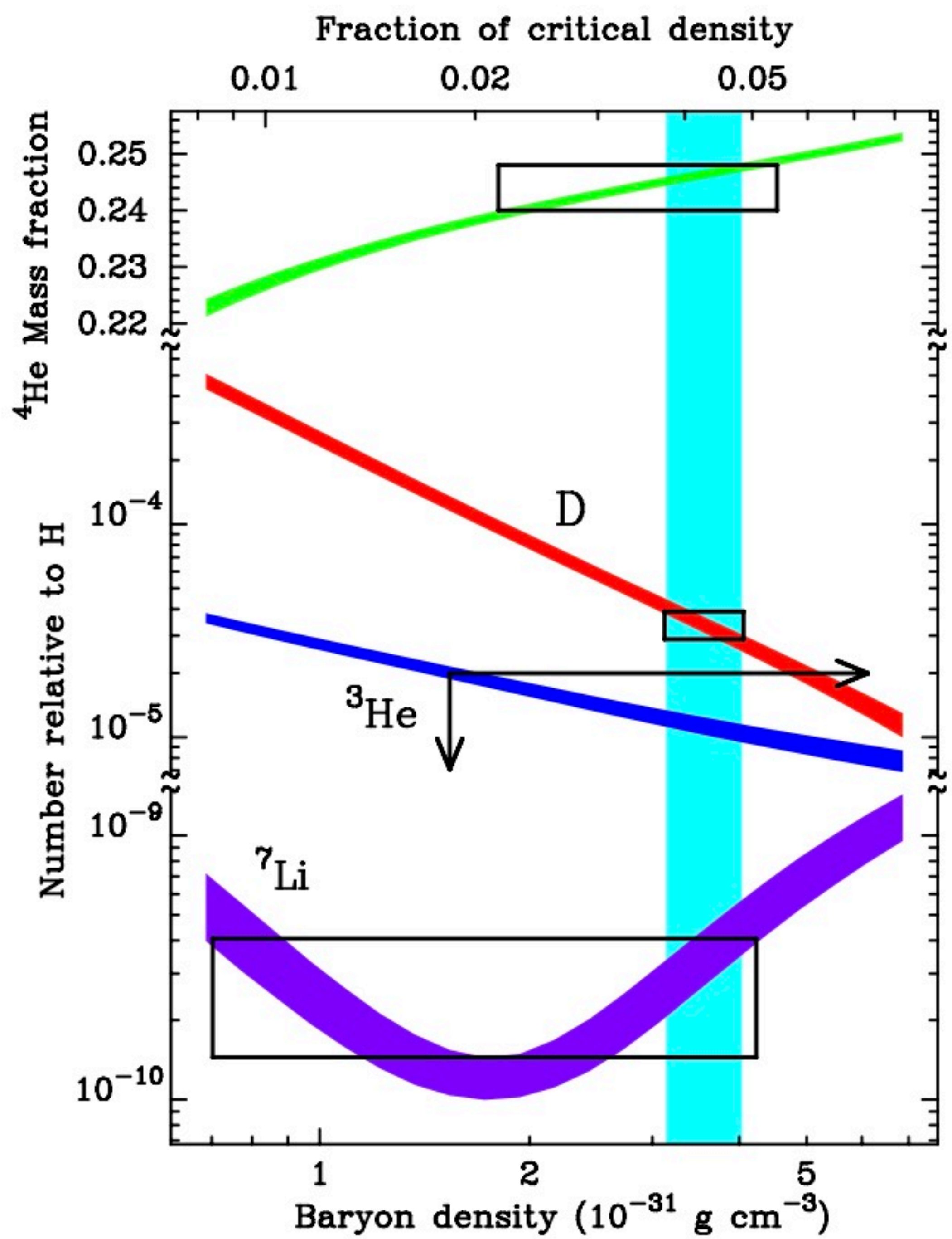
From here, assuming most neutrons end up in Helium, we can obtain an estimate for the primordial Helium / proton ratio

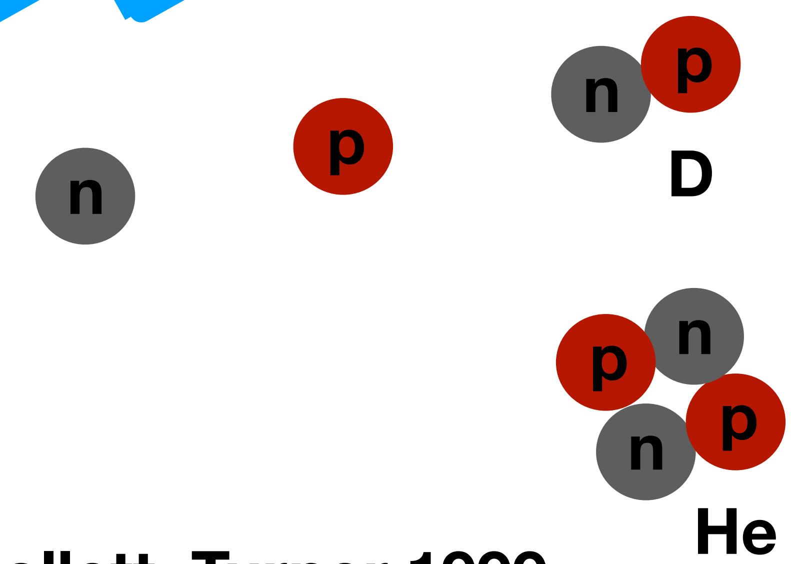
$$Y_p \equiv \frac{\rho_{^4\mathrm{He}}}{\rho_b} = \frac{4\,(n/2)}{n+p} = \frac{2n/p}{1+n/p} \approx \frac{1}{4}$$

BBN and hints at isotropy (iii)

Assume universe started as a hot dense, place







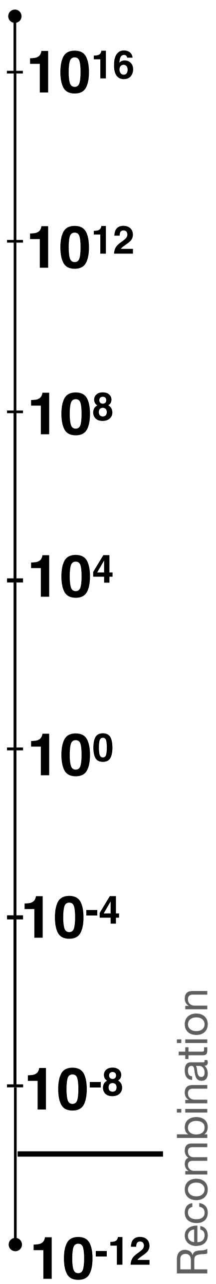
Burles, Nollett, Turner 1999

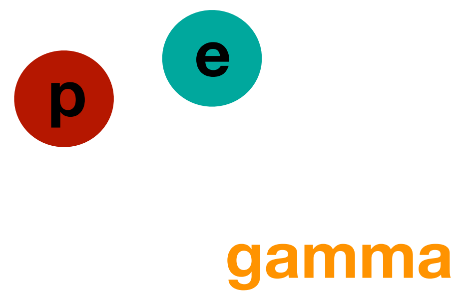
Blue range determined from deuterium measurements using (e.g.) quasar lines

$$Y_p \equiv \frac{\rho_{^4\mathrm{He}}}{\rho_b} = \frac{4\,(n/2)}{n+p} = \frac{2n/p}{1+n/p} \approx \frac{1}{4}$$

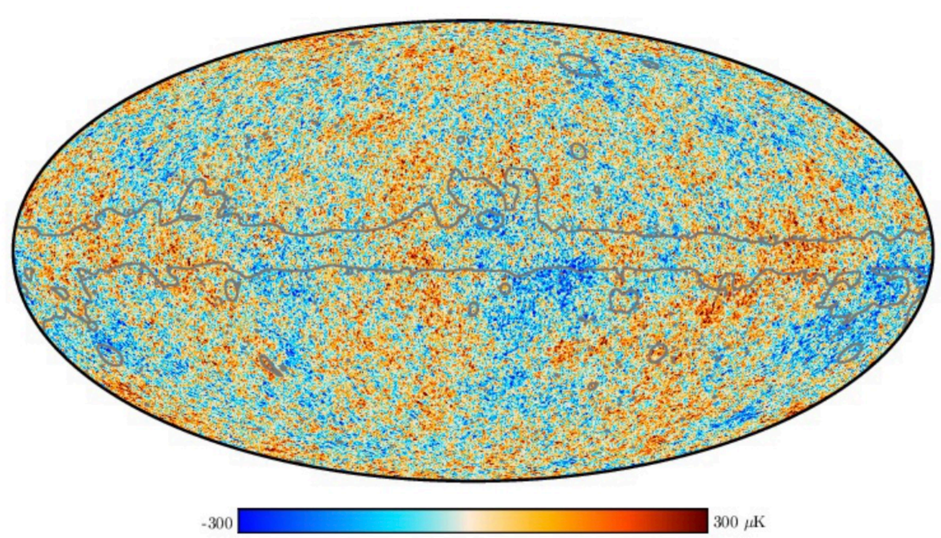
Recombination: photons from when the universe became transparent

At a certain temperature, photons would no longer have enough energy to split electrons off protons, and the universe would become neutral

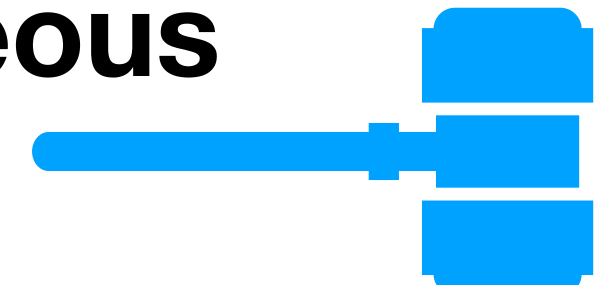


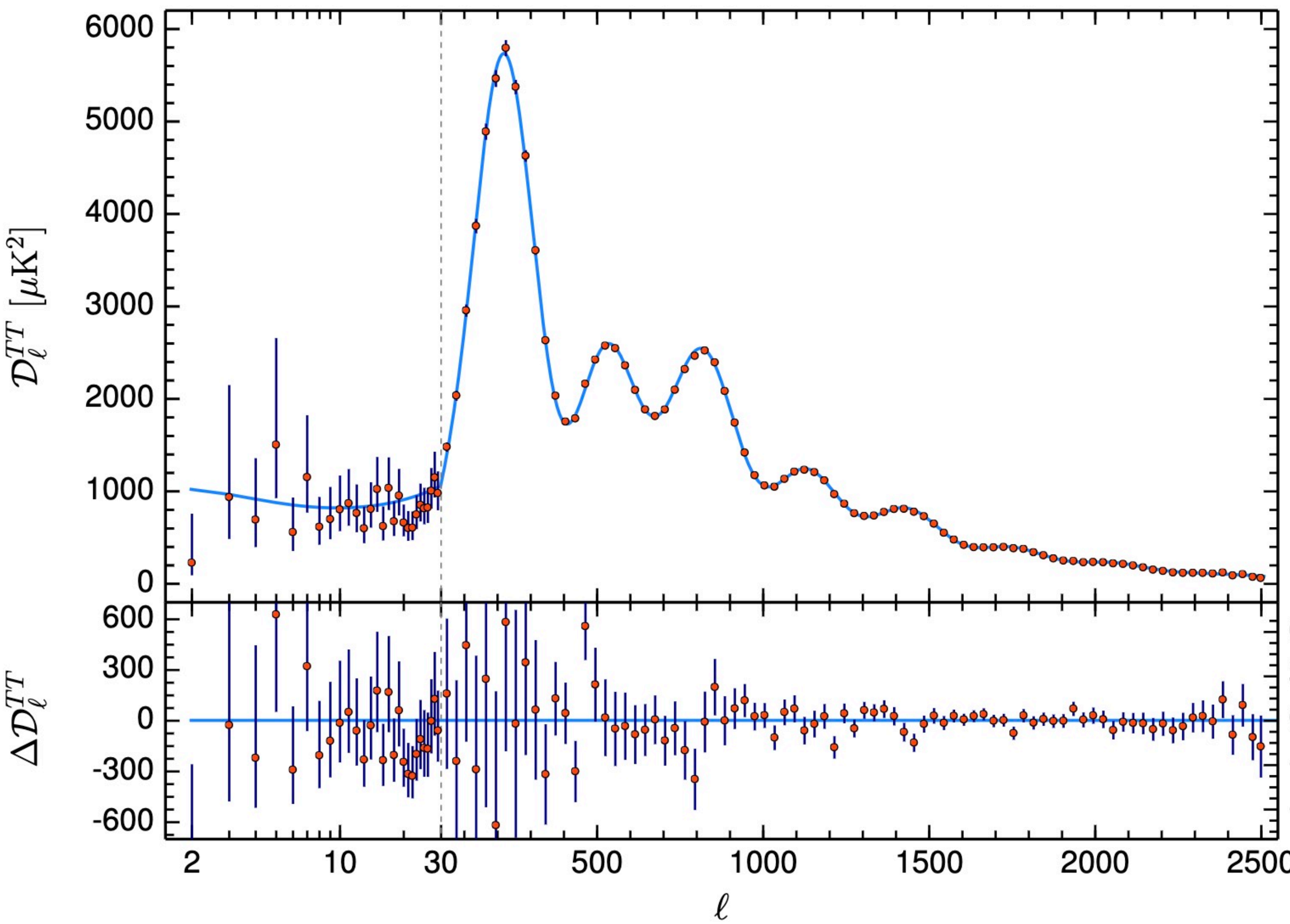


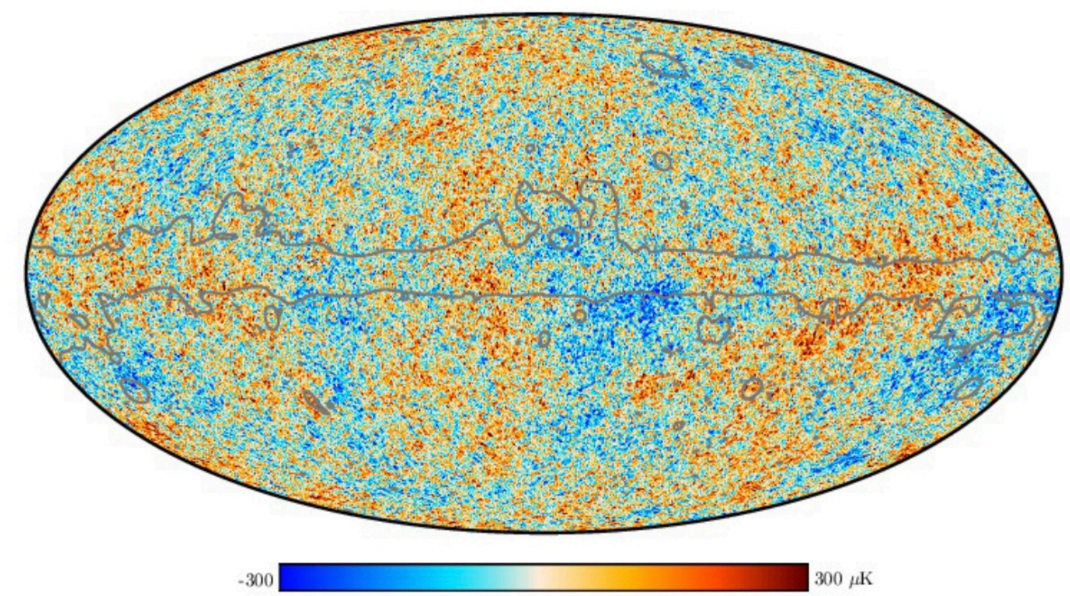
After this point, we should expect photons to stream freely. This background of photons is known as the Cosmic Microwave Background. It has been measured to rather exquisite precision by the COBE, WMAP, and PLANCK satellites. It is a blackbody with T~2.7 K and temperature variations $dT/T \sim 10^{-5}$



Ok so really, we're quite serious, our universe is rather isotropic and homogeneous







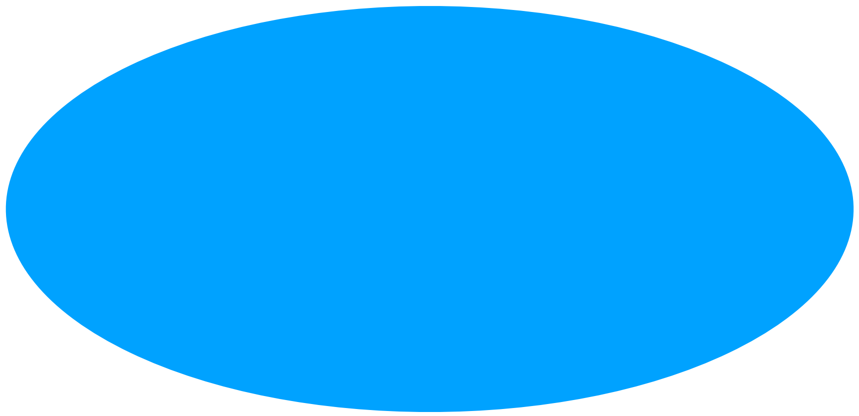
$$\left\langle a_{\ell m}^{T*}\, a_{\ell' m'}^{T} \right\rangle = C_\ell^{TT}\; \delta_{\ell'\ell}\, \delta_{m'm}$$

$$\delta T(\theta,\phi) = \sum_{\ell,m} a_{\ell m} Y_{\ell m}(\theta,\phi)$$

$$\mathcal{D}_\ell^{XY} = \frac{\ell(\ell+1)C_\ell^{XY}}{2\pi}$$

(Planck 2018)

So here is an isotropic, homogeneous model of the universe



$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu} \qquad G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$R_{\mu\nu} = \Gamma^{\alpha}{}_{\mu\nu,\alpha} - \Gamma^{\alpha}_{\mu\alpha,\nu} + \Gamma^{\alpha}{}_{\beta\alpha}\Gamma^{\beta}_{\mu\nu} - \Gamma^{\alpha}_{\beta\nu}\Gamma^{\beta}_{\mu\alpha}$$

$$\Gamma^{\mu}{}_{\alpha\beta} \equiv \frac{g^{\mu\nu}}{2}\left[g_{\alpha\nu,\beta} + g_{\beta\nu,\alpha} - g_{\alpha\beta,\nu}\right]$$

$$ds^2 = dt^2 - a^2(t)\, dx^2$$

$$T^{\mu}_{\ \nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -a^2 & & \\ & & -a^2 & \\ & & & -a^2 \end{pmatrix}$$

From these we obtain the Friedman equations

$$3m_{pl}^2 H^2 = \rho - \frac{3k}{a^2}$$

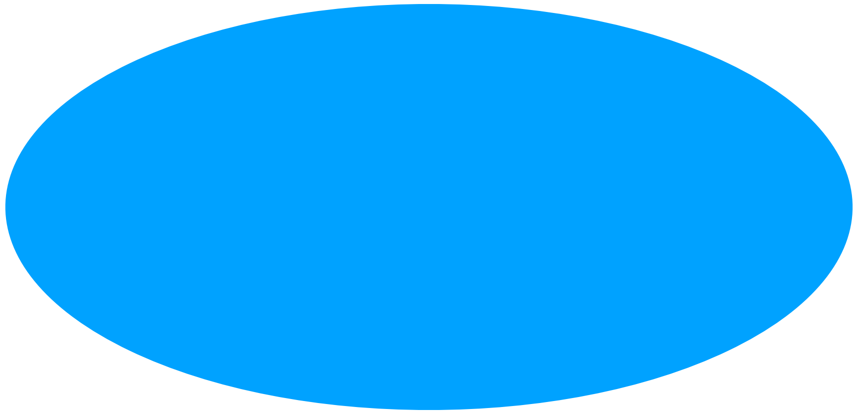
$$\dot{H} + H^2 = -\frac{1}{6m_{pl}^2}\left(\rho + 3p\right)$$

And from these the continuity equation

$$\frac{d\rho}{dt} + 3H(\rho + p) = 0$$

BOARD WORK

So here is an isotropic, homogeneous model of the universe



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So here is an isotropic, homogeneous model of the universe

From the continuity equation we obtain

$$\frac{d\rho}{dt} + 3H(\rho + p) = 0$$

$$\frac{d\ln\rho}{d\ln a} = -3(1+w) \qquad w \equiv \frac{p}{\rho}$$

Using the first Fried. Eq. the energy density scales as

$$\rho \propto a^{-3(1+w)} \qquad a(t) \propto \begin{cases} t^{2/3(1+w)} & w \neq -1 \\ e^{Ht} & w = -1 \end{cases}$$

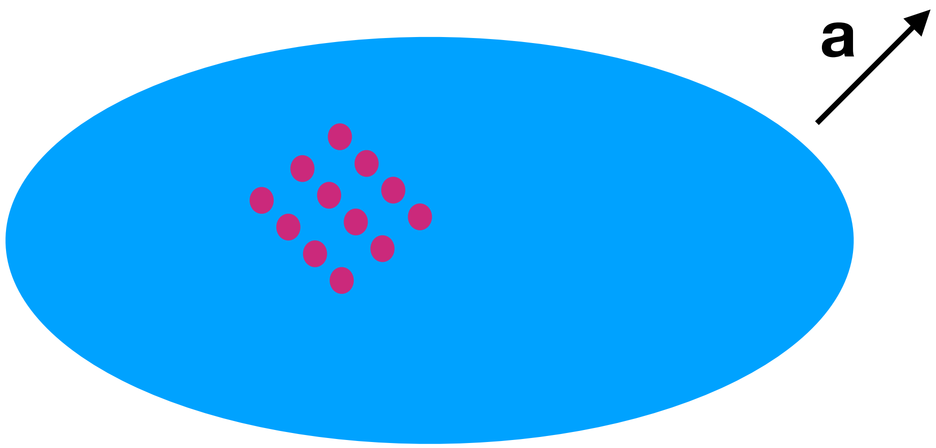
$\rho = \rho_0 a^{-3}$ matter

$\rho = \rho_0 a^{-4}$ radiation

$\rho = \rho_0$ vacuum energy

	w	$\rho(a)$	$a(t)$
MD	0	a^{-3}	$t^{2/3}$
RD	$\frac{1}{3}$	a^{-4}	$t^{1/2}$
Λ	-1	a^0	e^{Ht}

Stuff in an isotropic universe



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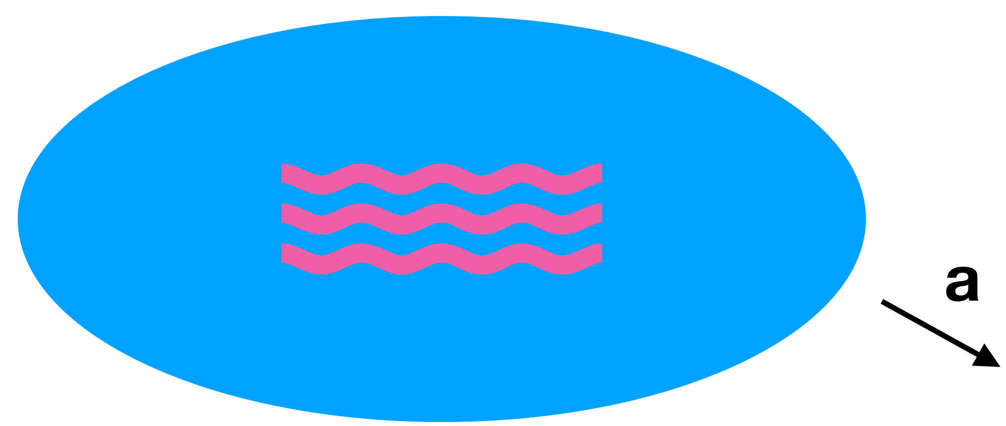
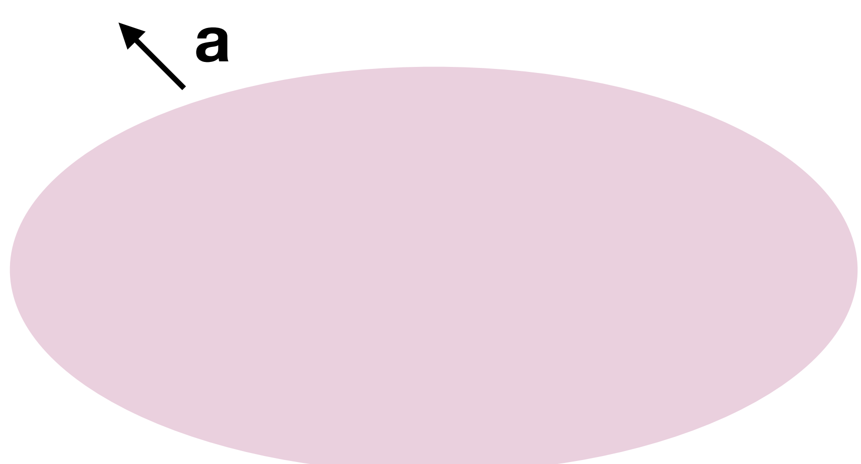
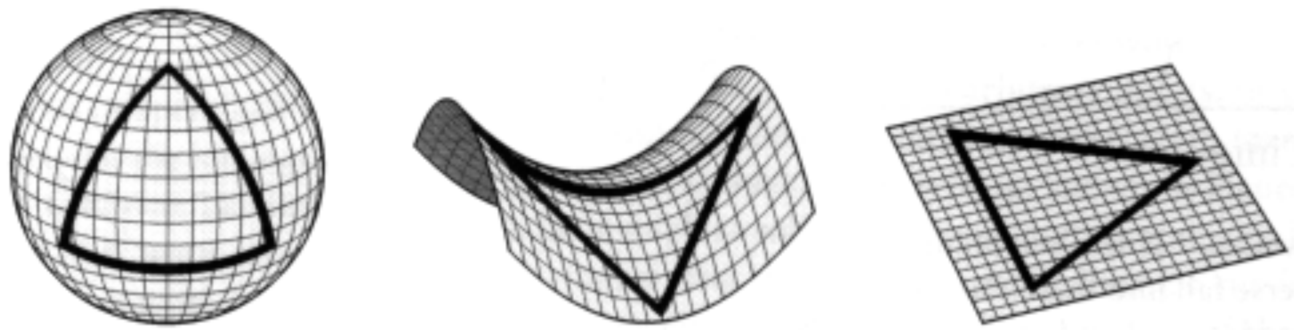




$\rho = \rho_0$ vacuum energy

Critical density

How much energy density resides in the curvature of space, as whole?



Define a sum over energy densities (photons, baryons, electrons, neutrinos, DM, DE…)

$$\rho \equiv \sum_i \rho_i \qquad \rho_{crit} \equiv 3H^2 m_{pl}^2 \qquad \Omega_i \equiv \frac{\rho_0^i}{\rho_{\rm crit}}$$

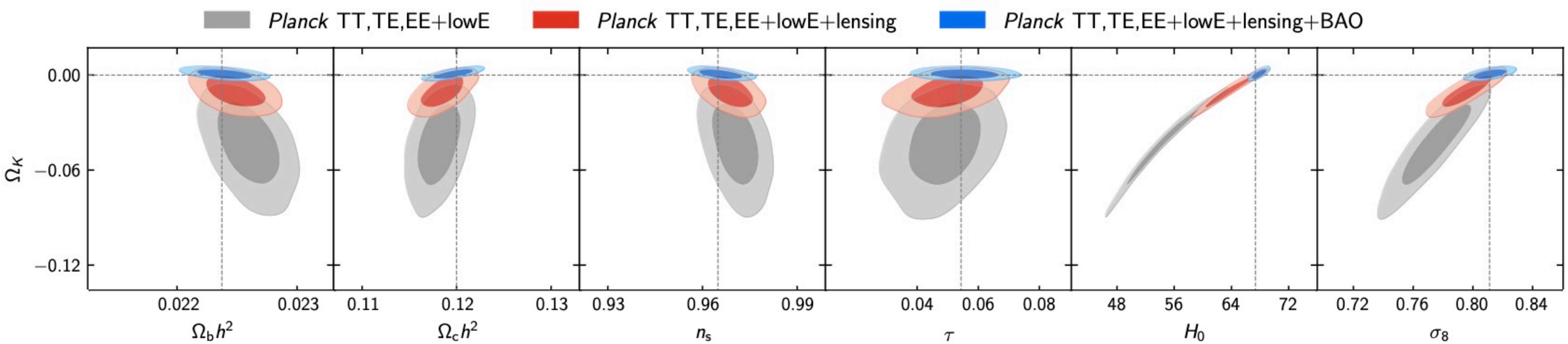
$$\Omega_k \equiv -k/a_0^2 H_0^2$$

Friedman Eq. for present day "0" time energy densities including curvature

$$\left(\frac{H}{H_0}\right)^2 = \sum_i \Omega_i a^{-3(1+w_i)} + \Omega_k a^{-2} \qquad \sum_i \Omega_i + \Omega_k = 1$$

Critical density: we are very critical

$$\Omega_k \approx 0$$



Planck 2018

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Big bang cosmology puzzle 1: horizon

Consider the causal distance that light can travel in an expanding universe

$$d\tau \equiv c\frac{dt}{a}$$

Big bang cosmology puzzle 1: horizon

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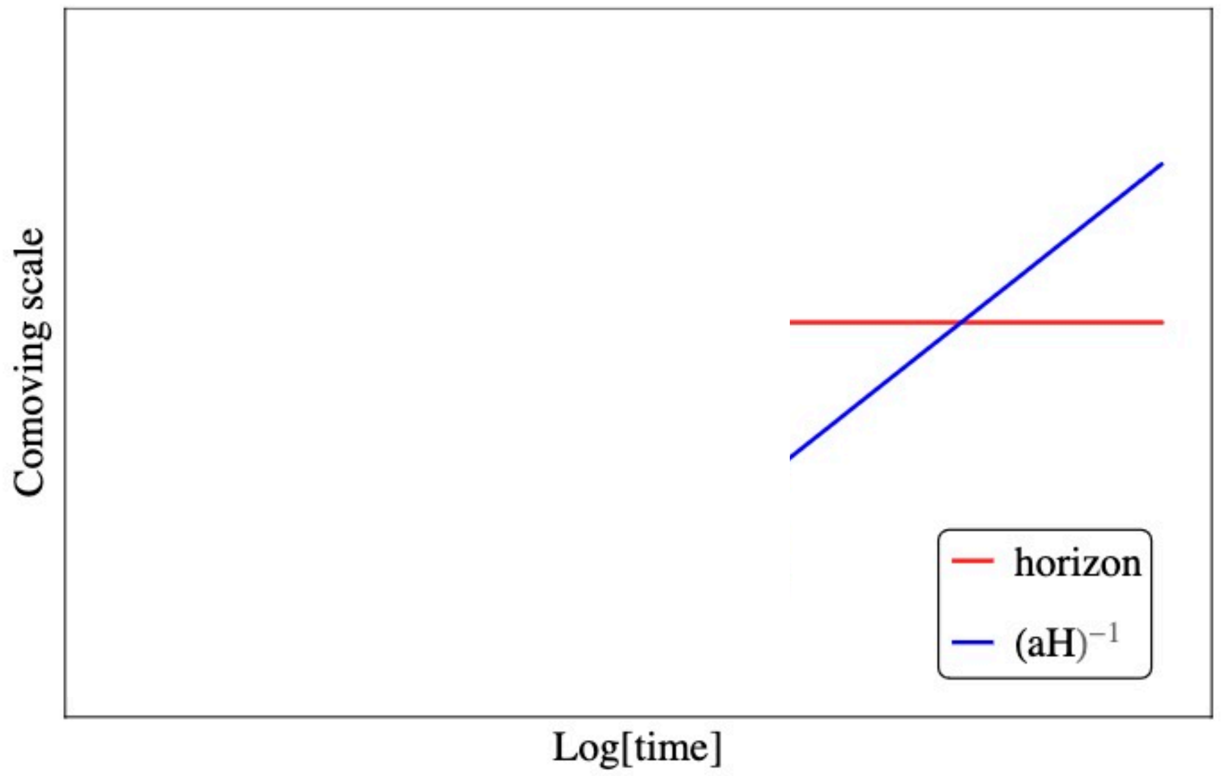
Big bang cosmology puzzle 1: horizon

Consider the causal distance that light can travel in an expanding universe

$$d\tau \equiv c\frac{dt}{a}$$

$$\tau \equiv \int_0^t \frac{\mathrm{d}t'}{a(t')} = \int_0^a \frac{\mathrm{d}a}{Ha^2} = \int_0^a \mathrm{d}\ln a \left(\frac{1}{aH}\right)$$

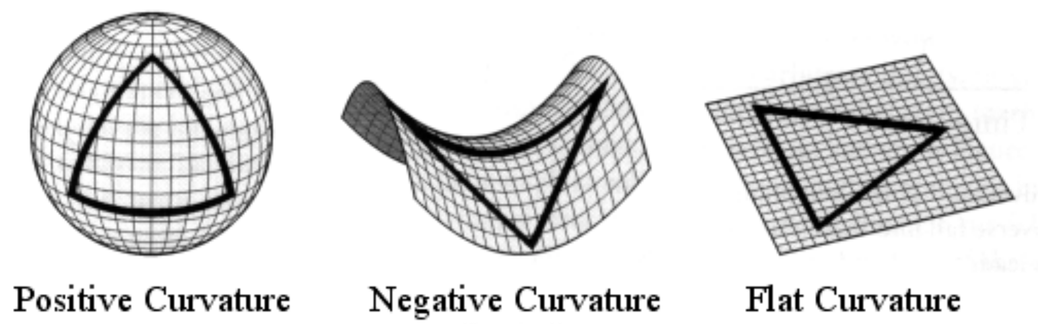
$$\tau = \int_0^a \frac{\mathrm{d}a}{Ha^2} \propto \begin{cases} a & \mathrm{RD} \\ a^{1/2} & \mathrm{MD} \end{cases}$$



The universe is isotropic and homogeneous - but how?
None of it would have been in causal contact, given only rad/matter dominated expansion.

Big bang cosmology puzzle 2: flatness

A similar logic applies to the flatness of our universe



The Friedmann equation can be re-expressed

$$3m_{pl}^2 H^2 = \rho - \frac{3k}{a^2} \qquad 1 - \Omega(a) = -\frac{k}{(aH)^2} \qquad \Omega(a) \equiv \frac{\rho}{\rho_{crit}}$$

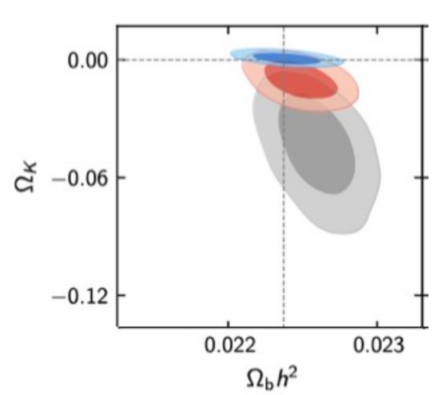
Where again we know that during rad/matter expansion

$$(aH)^{-1} = H_0^{-1} a^{\frac{1}{2}(1+3w)}$$

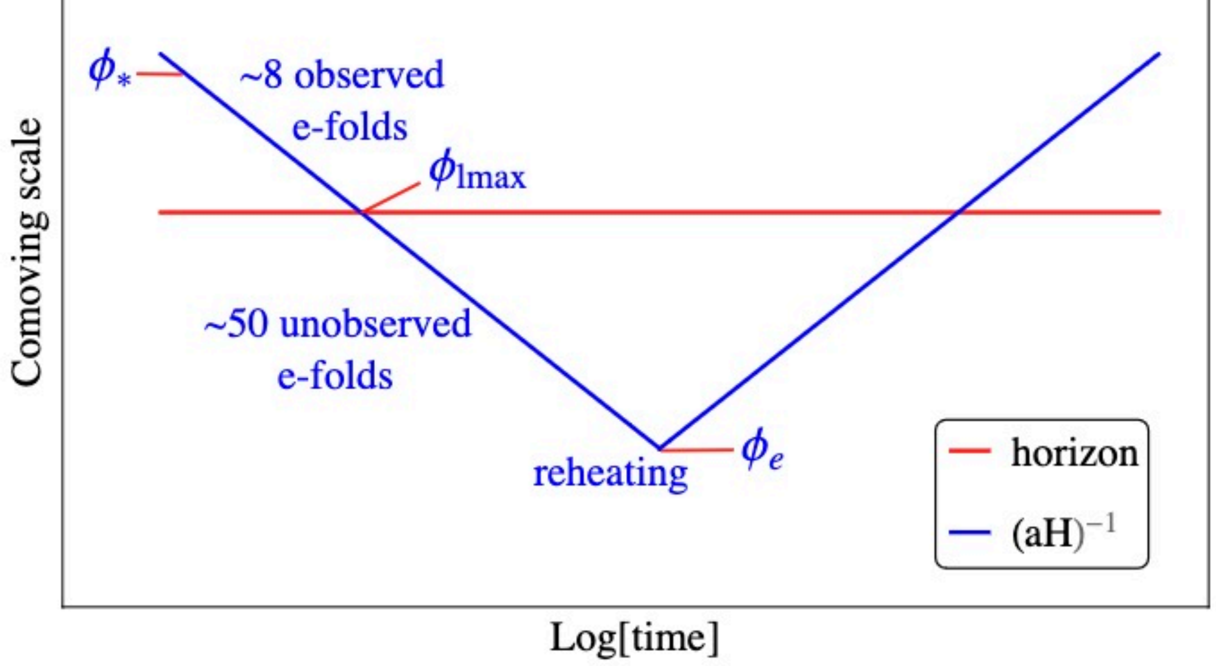
But we've measured the RHS to be $\sim 0 \pm 0.02$

Fine tuning?

$$|\Omega(a_{\rm BBN}) - 1| \leq \mathcal{O}(10^{-16})$$
$$|\Omega(a_{\rm GUT}) - 1| \leq \mathcal{O}(10^{-55})$$



Inflation



Both the horizon and flatness features can be explained by a period of cosmic inflation. The coming horizon $(aH)^{-1}$ shrinks before it grows during rad/matter domination. So we require

$$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0 \quad \Rightarrow \quad \frac{d^2a}{dt^2} > 0$$

The second Friedmann equation then implies we need $p < -\rho/3$

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p)$$

In fact, we've already seen what's required for this - vacuum energy domination implies accelerated expansion of the universe

Scalar field in FRW

The extended Einstein-Hilbert action for a scalar field

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right]$$

In an FRW metric this yields an energy-momentum tensor and density/pressure components of the form

$$T^{(\phi)}_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_\phi}{\delta g^{\mu\nu}} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial^\sigma\phi\partial_\sigma\phi + V(\phi)\right)$$

assume $\phi(x,t) = \phi(t)$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

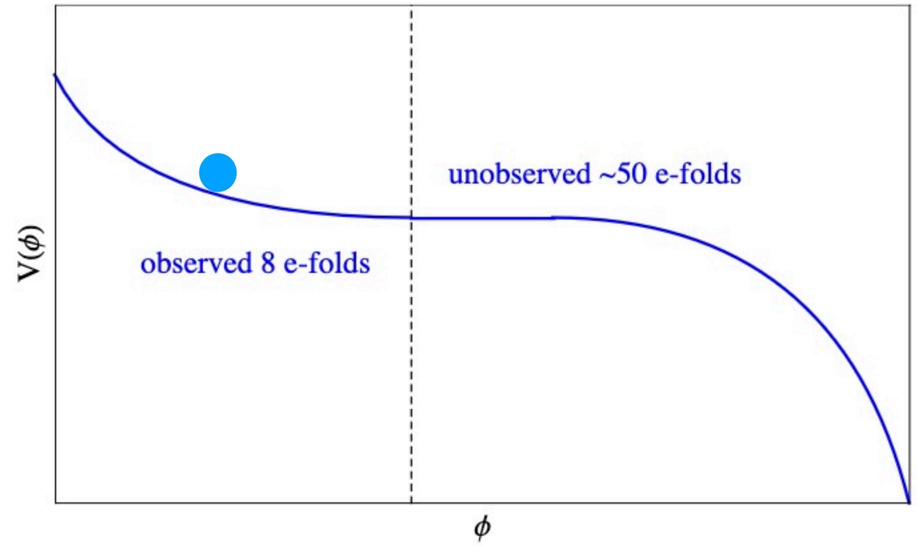
$$w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V}$$

From this we immediately see that w=-1 is obtained for V >> $\dot{\phi}$

Varying the action leads to EOM that also illuminate how a scalar field can maintain a fixed energy density

$$\frac{\delta S_\phi}{\delta\phi} = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\partial^\mu\phi) + V_{,\phi} = 0$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$



Inflation from a scalar field in FRW

$$w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V}$$

So to summarize, the conditions for scalar field inflation are

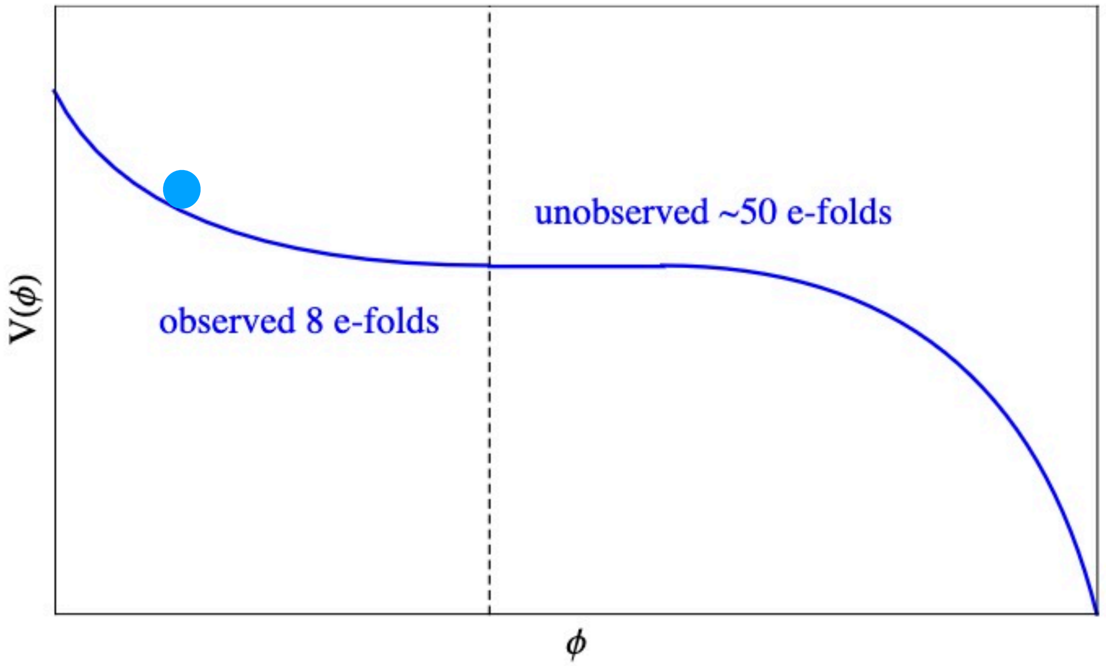
$$\dot{\phi}^2 \ll V(\phi)$$

To yield the correct EOS, along with

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

$$|\ddot{\phi}| \ \ll \ |3H\dot{\phi}| \, , \, |V_{,\phi}|$$

To ensure that the scalar field remains stationary. These can be expressed as conditions on the “slow-roll” potential of the scalar field, where both these terms are less than 1



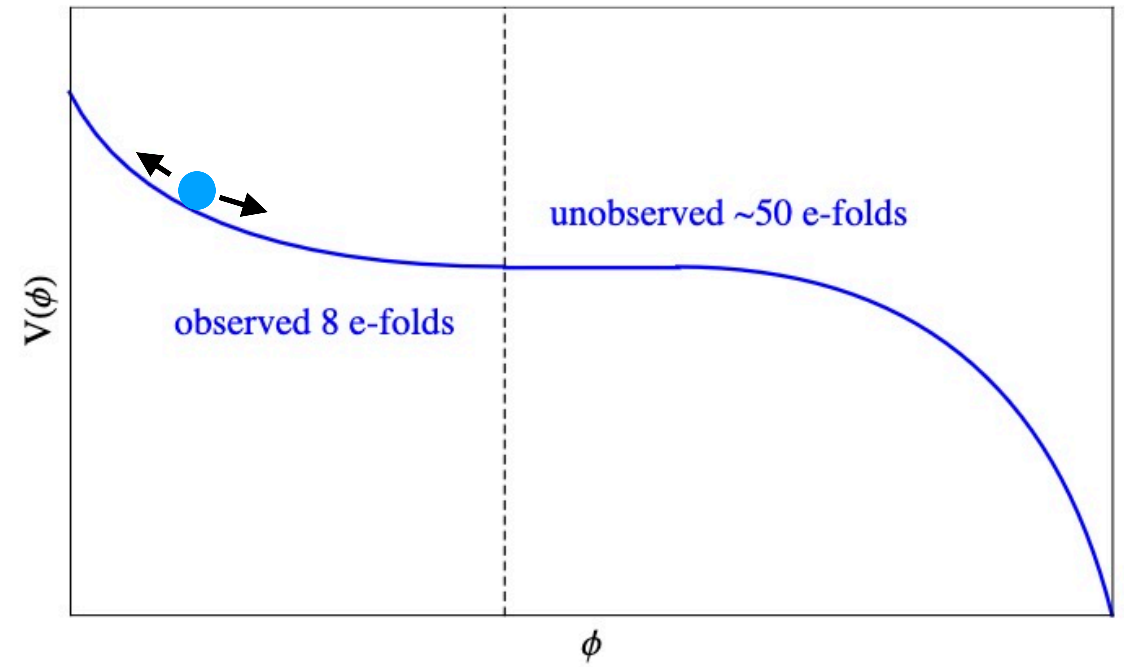
$$\epsilon_{\rm v}(\phi) \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 < 1$$

$$\eta_{\rm v}(\phi) \equiv M_{\rm pl}^2 \frac{V_{,\phi\phi}}{V} < 1$$

So long as these conditions are satisfied,

Scalar and tensor perturbations, ρ_i

Quantum fluctuations of the inflaton during inflation lead to scalar and tensor perturbations visible on the CMB and in matter overdensities that will come to form galaxies.



$$\left(\frac{dT}{T}\right)^2 \propto A_s = \frac{V}{24\pi^2 m_{\rm Pl}^4 \epsilon_V}$$

$$A_t = \frac{2V}{3\pi^2 m_{\rm Pl}^4}$$

Notice that the tensor perturbation amplitude only depends on V. So far, Planck has set a limit on tensor perturbations, and measured scalar perturbations, leading to a bound on the energy density during inflation

$$r \equiv \frac{A_t}{A_s} \qquad V_*^{1/4} \lesssim 2 \times 10^{16}\ \mathrm{GeV} \left(\frac{r}{0.1}\right)^{1/4}$$

Inflation from a scalar field in FRW

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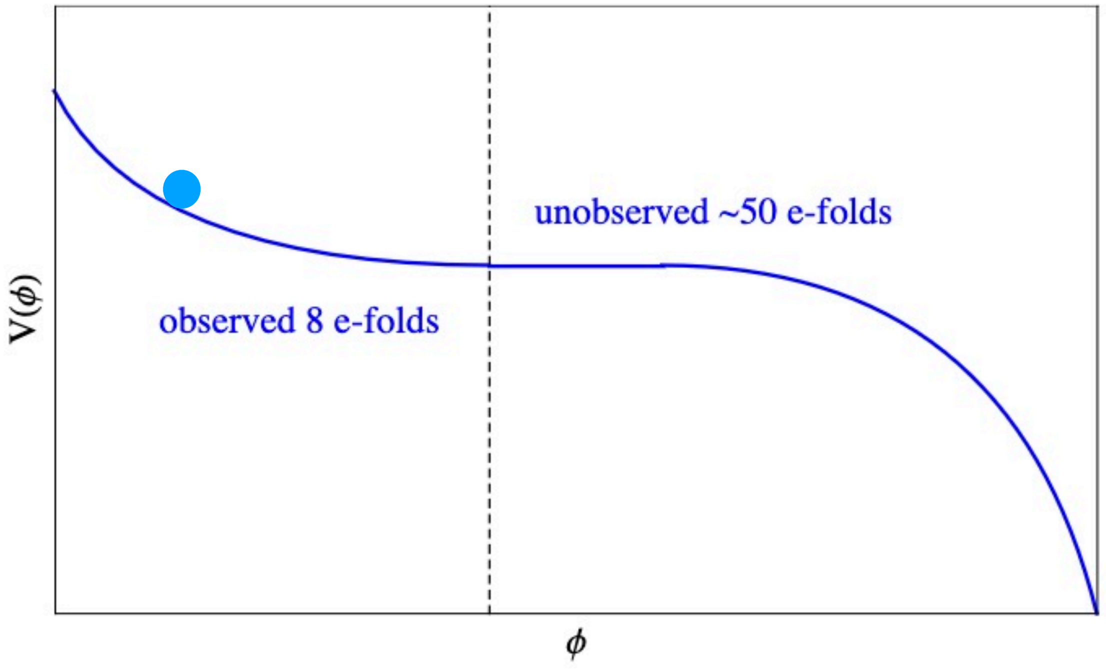

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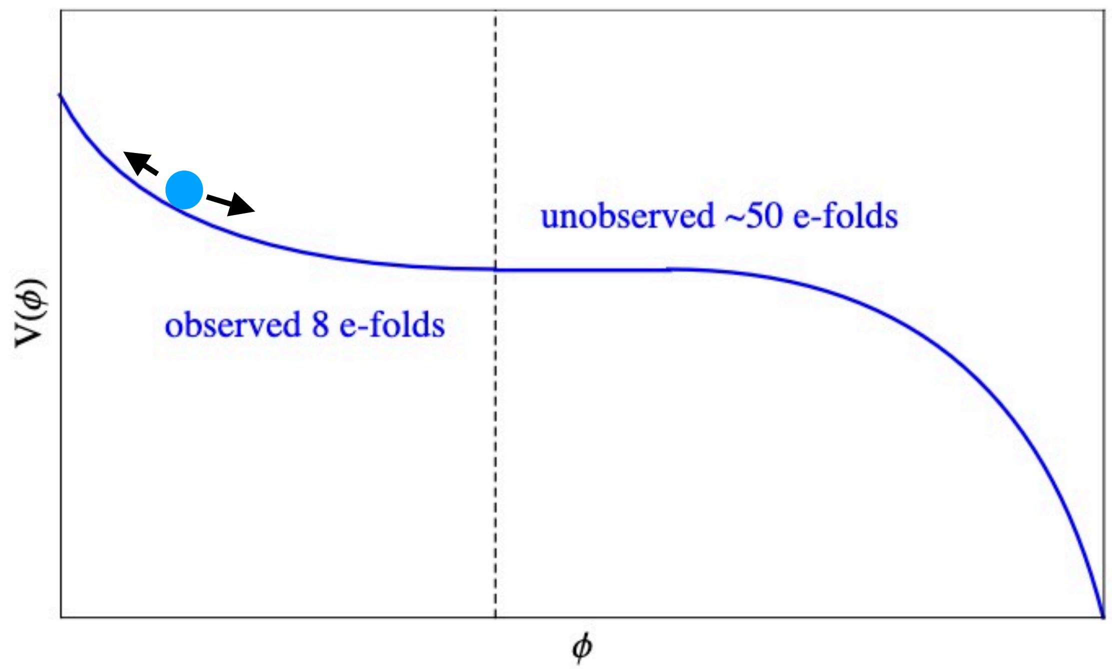
Number of e-folds and the Lyth bound

Assuming the universe expands with a matter/radiation EOS after inflation a certain number of e-folds of expansion are required to satisfy a connected horizon and flatness

$$a(t) \sim e^{Ht}$$ **}e-folds, N**

$$N = \int_{t_*}^{t_e} H \mathrm{d}t = m_{\mathrm{Pl}}^2 \int_{\phi_e}^{\phi_*} \frac{V}{V_\phi} \mathrm{d}\phi \sim 40 - 60$$



Combining this expression with the scalar and tensor perturbations leads to a bound on the field transit of a single inflaton during inflation, called the **Lyth Bound**

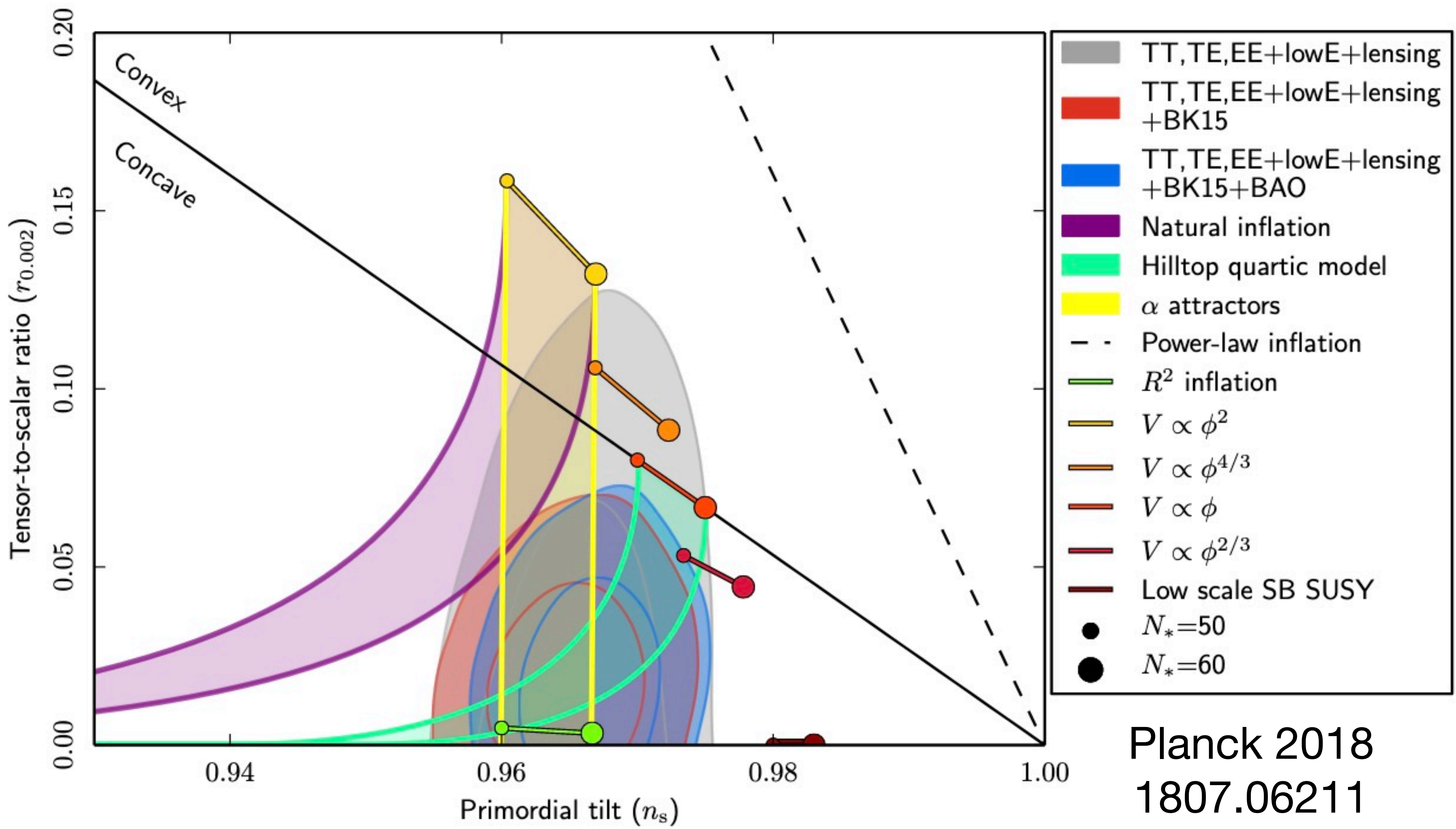
$$\frac{\Delta\phi}{m_{\mathrm{Pl}}} \gtrsim N\sqrt{\frac{r}{8}} = N\sqrt{2\epsilon_V} \qquad A_t = \frac{2V}{3\pi^2 m_{\mathrm{Pl}}^4} \qquad A_s = \frac{V}{24\pi^2 m_{\mathrm{Pl}}^4 \epsilon_V} \qquad r \equiv \frac{A_t}{A_s}$$

This has interesting implications for Planck-scale physics

Current realistic inflaton models

Final inflaton parameter: spectral index quantifies deviation from scale-invariant power spectrum

$$\boxed{n_s} = 1 + 2\eta_V^\star - 6\epsilon_V^\star$$



Planck 2018
1807.06211

Hilltop quartic model	$\Lambda^4\left(1-\phi^4/\mu_4^4+\ldots\right)$	$-2<\log_{10}(\mu_4/M_{\rm Pl})<2$	−0.3	−1.4
D-brane inflation ($p=2$)	$\Lambda^4\left(1-\mu_{D2}^2/\phi^p+\ldots\right)$	$-6<\log_{10}(\mu_{D2}/M_{\rm Pl})<0.3$	−2.0	0.6
D-brane inflation ($p=4$)	$\Lambda^4\left(1-\mu_{D4}^4/\phi^p+\ldots\right)$	$-6<\log_{10}(\mu_{D4}/M_{\rm Pl})<0.3$	−3.5	−0.4
Potential with exponential tails	$\Lambda^4\left[1-\exp\left(-q\phi/M_{\rm Pl}\right)+\ldots\right]$	$-3<\log_{10}q<3$	−0.4	−1.0
Spontaneously broken SUSY	$\Lambda^4\left[1+\alpha_h\log\left(\phi/M_{\rm Pl}\right)+\ldots\right]$	$-2.5<\log_{10}\alpha_h<1$	6.7	−6.8
E-model ($n=1$)	$\Lambda^4\left\{1-\exp\left[-\sqrt{2}\phi\left(\sqrt{3\alpha_1^E}M_{\rm Pl}\right)^{-1}\right]\right\}^{2n}$	$-2<\log_{10}\alpha_1^E<4$	0.8	−0.3
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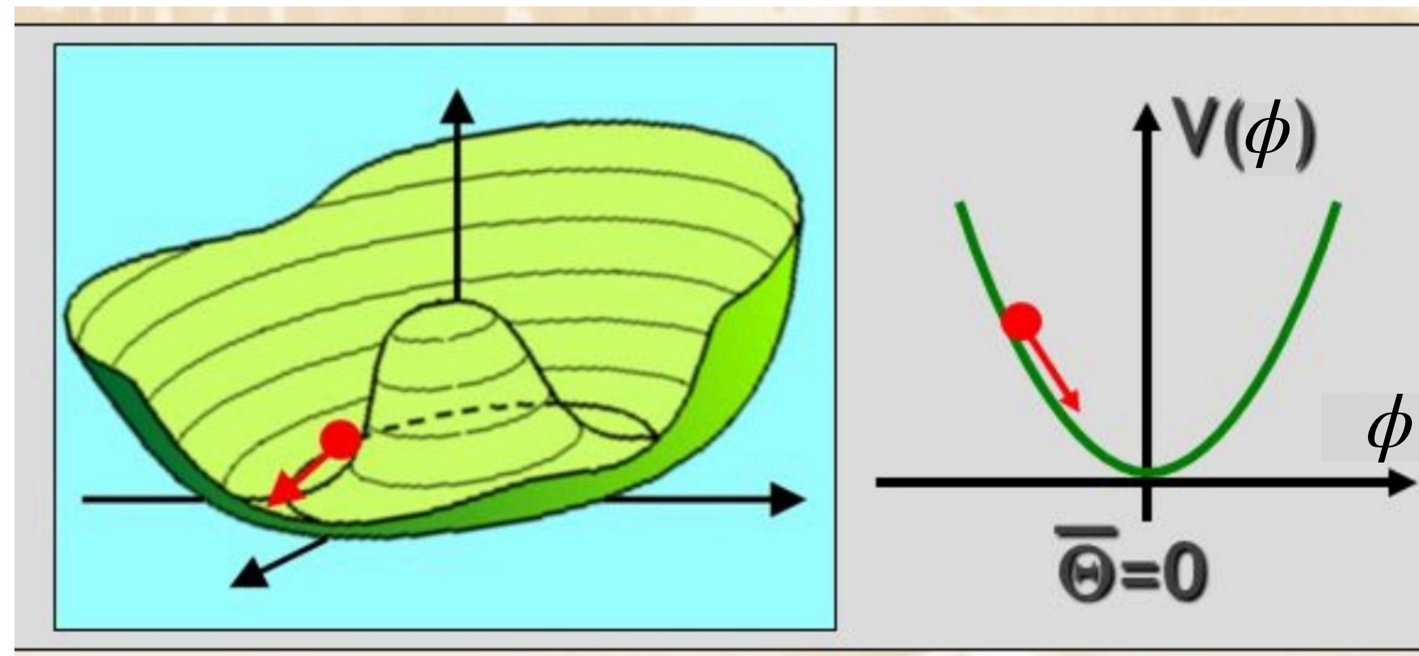
Bosonic dark matter (axion, alp, any ultralight)

“the misalignment mechanism”

$$w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V}$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$



from Raffelt

Consider an inflaton or another scalar field in an FRW background with

$$V = \frac{1}{2}m^2\phi^2$$

With H > m, this field will act like an overdamped harmonic oscillator, however once H << m, the field will have a simpler EOM.

1. Find the time-average EOS for this field. Assume the field starts at $\phi = \phi_0 \neq 0$.
2. What is the average energy density of this scalar field at the time it starts oscillating?

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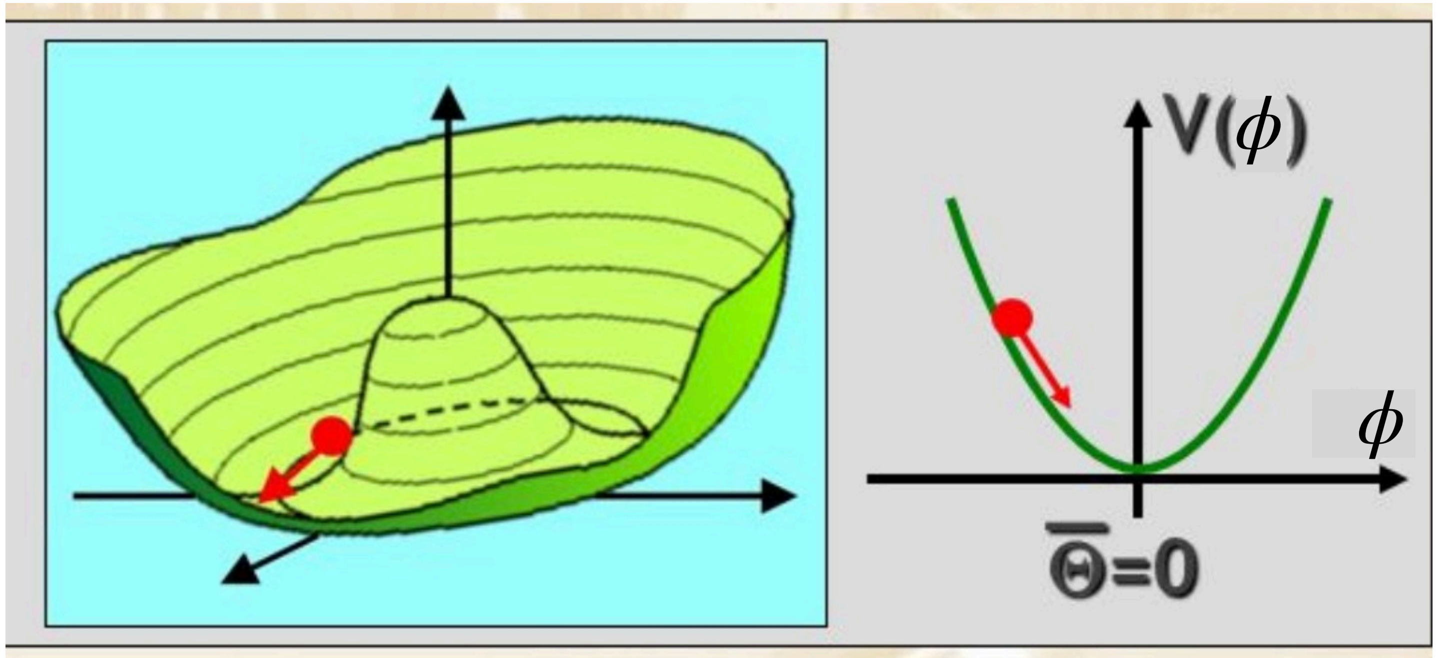

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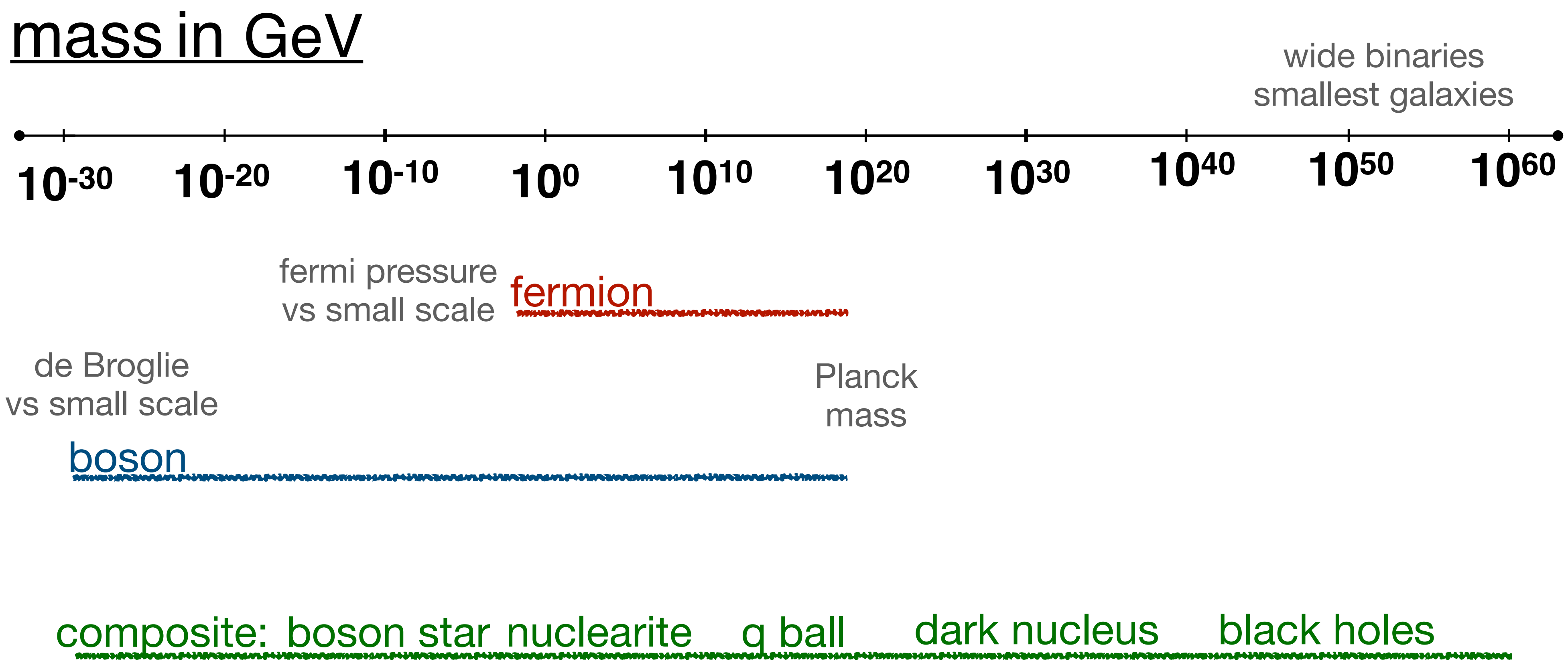
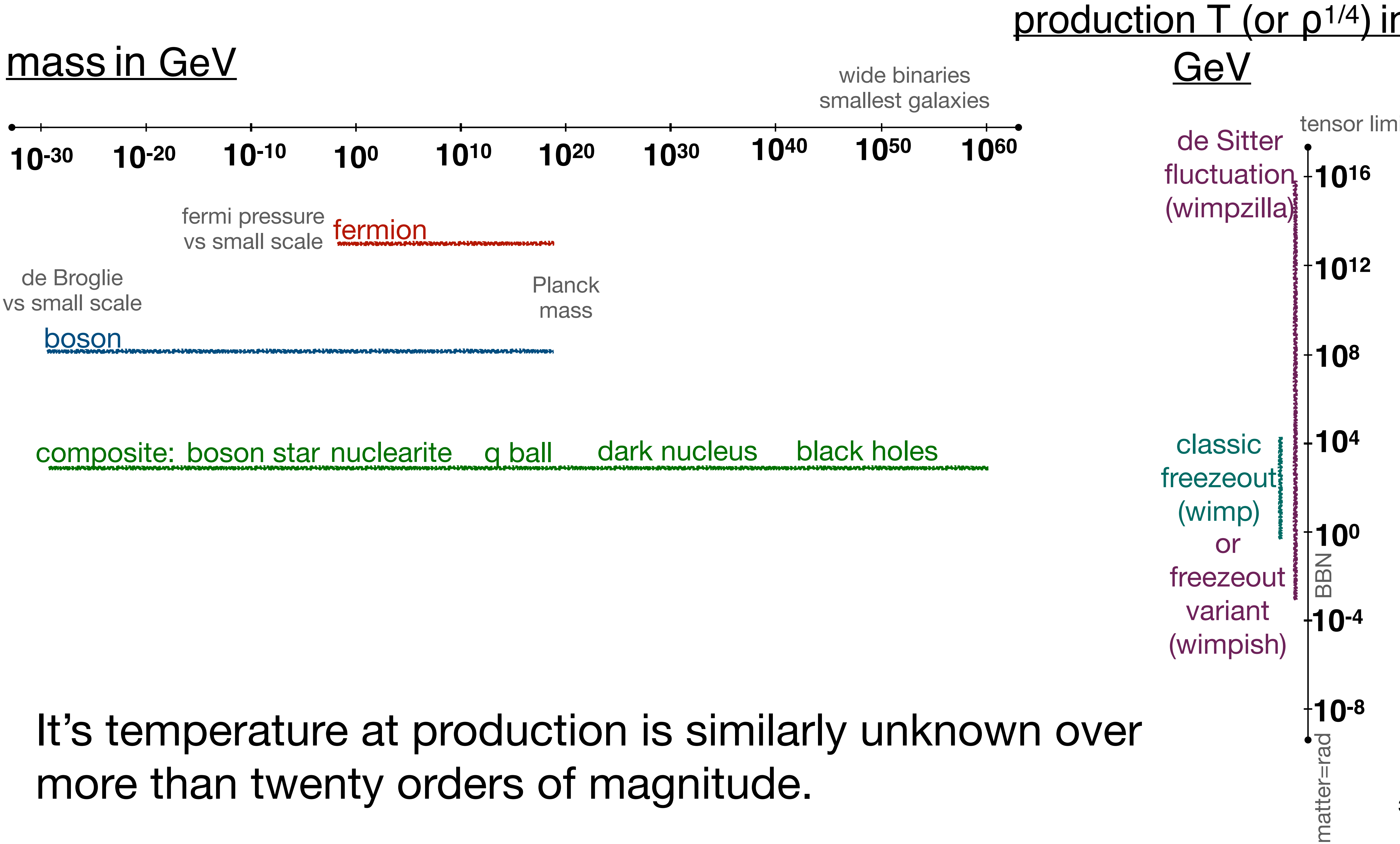
w=0, $\rho \sim m^2\phi_0^2$

What do we know about dark matter?



Its mass falls along a wide range, for fermions, bosons, or composite dark matter the allowed masses are different.

What do we know about dark matter?



It's temperature at production is similarly unknown over more than twenty orders of magnitude.

Energy and entropy densities

The energy density and pressure for generalized fields are given by

$$\rho = \frac{g}{2\pi^2}\int_m^\infty \frac{(E^2-m^2)^{1/2}}{e^{(E-\mu)/T}\pm 1}E^2\,dE \qquad p = \frac{g}{6\pi^2}\int_m^\infty \frac{(E^2-m^2)^{3/2}}{e^{(E-\mu)/T}\pm 1}E^2\,dE$$

Where g counts over the spin states of the field and the ± is for bosons and fermions, respectively. Integrating in the limit that T >> m, μ

$$\rho(T) = \begin{cases} \frac{\pi^2}{30} g T^4 & \text{for bosons,} \\ \frac{7}{8}\frac{\pi^2}{30} g T^4 & \text{for fermions} \end{cases}$$

Defining the entropy density for a radiation bath with w=1/3, we find

$$s \equiv \frac{S}{V} = \frac{\rho + p}{T} = \frac{4\rho}{3T} \qquad \begin{cases} \rho(T) = \frac{\pi^2}{30} g_* T^4, \\ s(T) = \frac{4}{3}\frac{\pi^2}{30} g_{*s} T^3 \end{cases}$$

Where we define the number of degrees of freedom as

$$g_* = \sum_{i=bosons} g_i \left(\frac{T_i}{T_\gamma}\right)^4 + \frac{7}{8}\sum_{i=\text{ fermions}} g_i \left(\frac{T_i}{T_\gamma}\right)^4$$

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in the SM at high T $g_* \sim 107$

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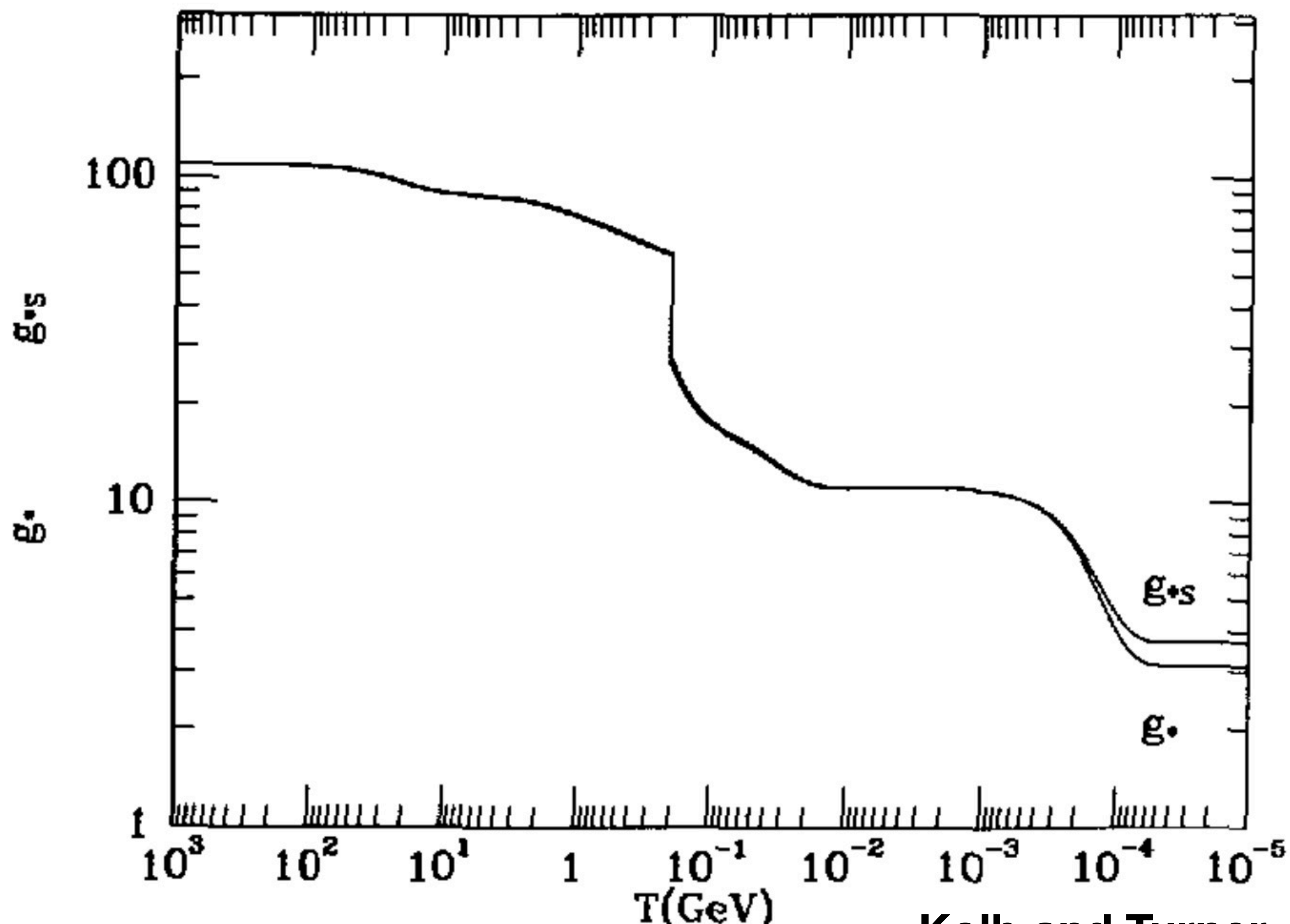

Kolb and Turner

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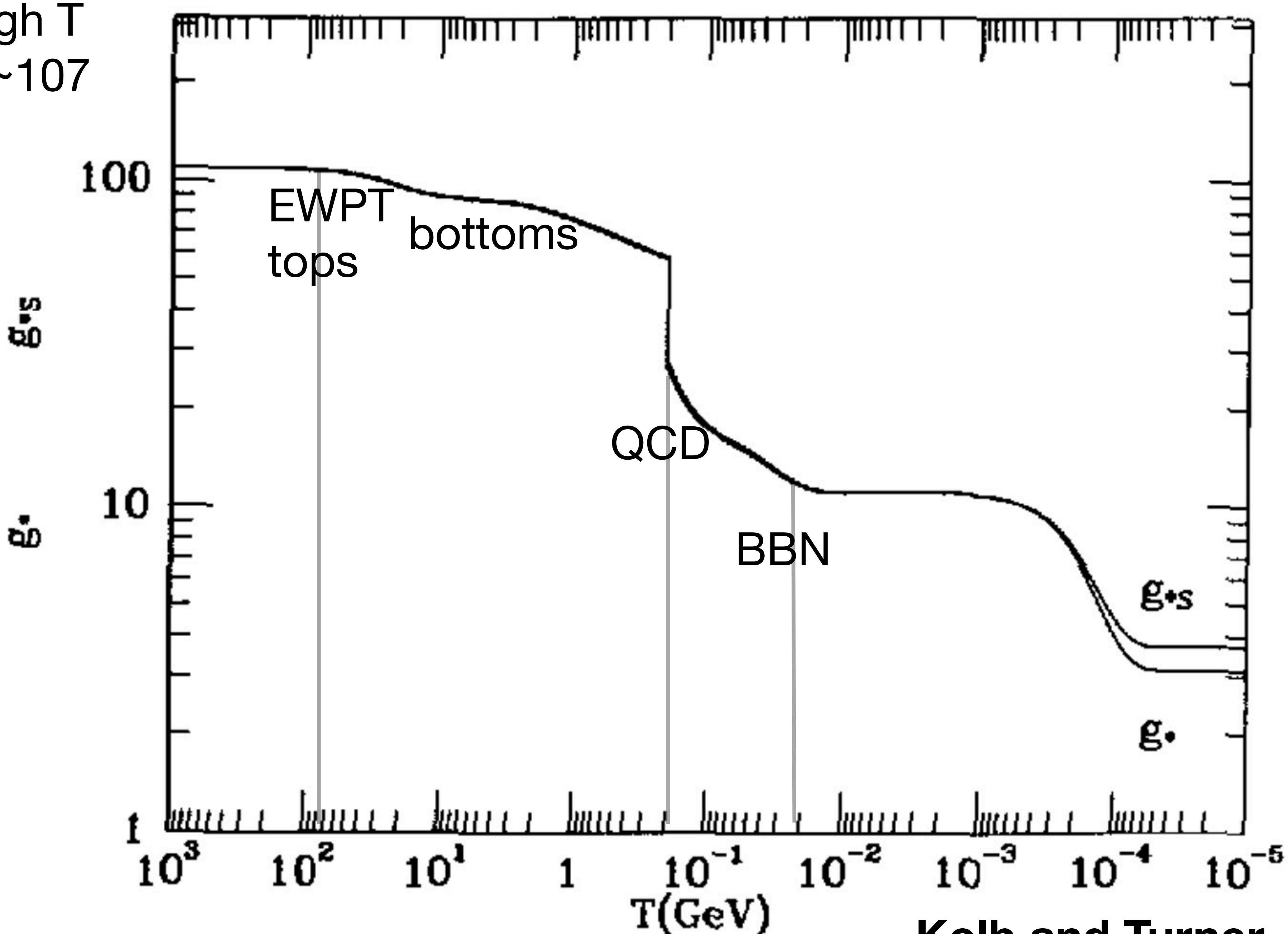
in the SM at high T $g_* \sim 107$

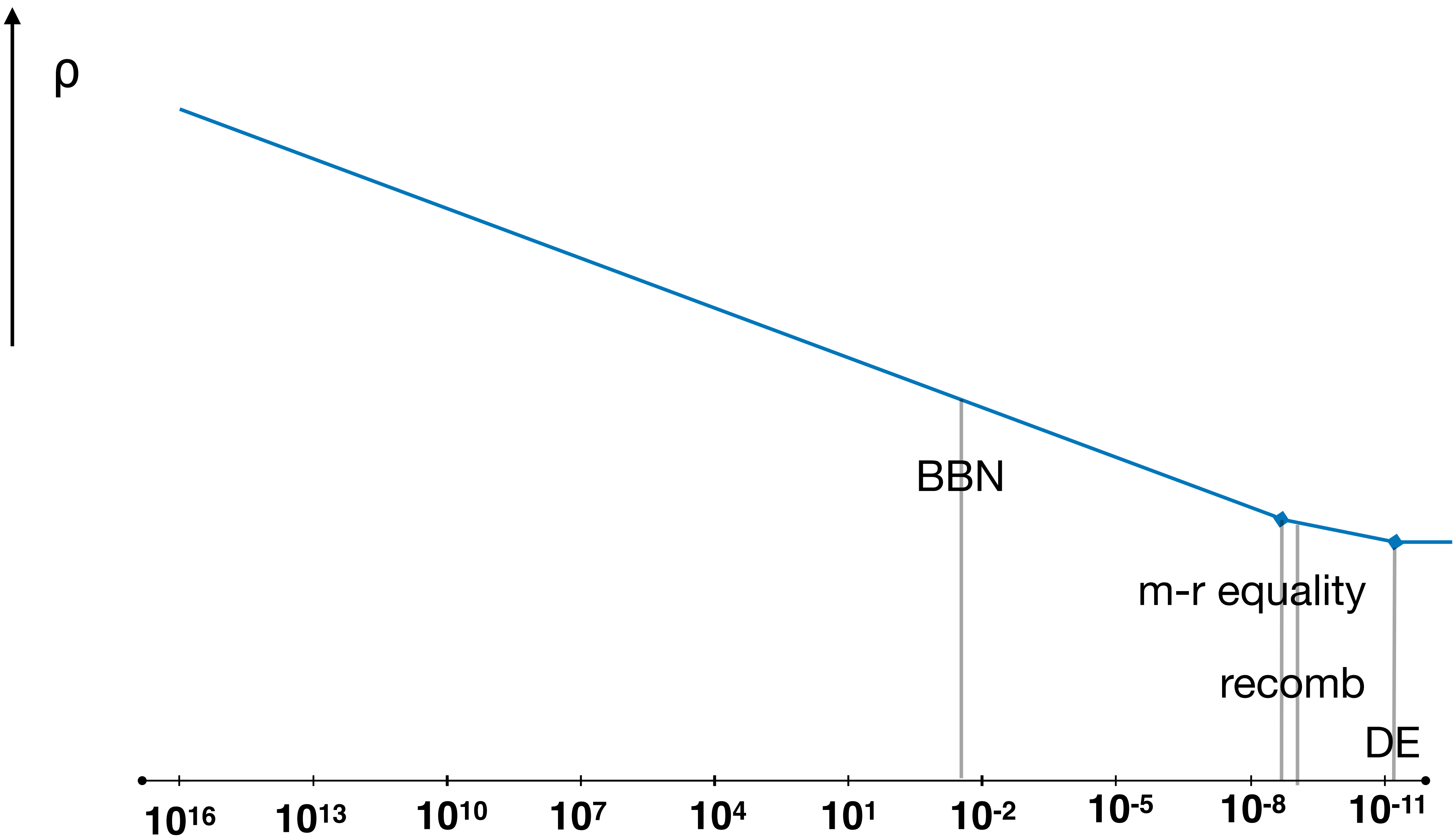


Kolb and Turner

in the SM at low T $g_* \sim 3$

Building a universe

The universe, assuming a radiation-dominated phase of expansion prior to BBN



$$T(\text{or } \rho^{1/4}) \propto \frac{1}{a} \text{ in GeV}$$

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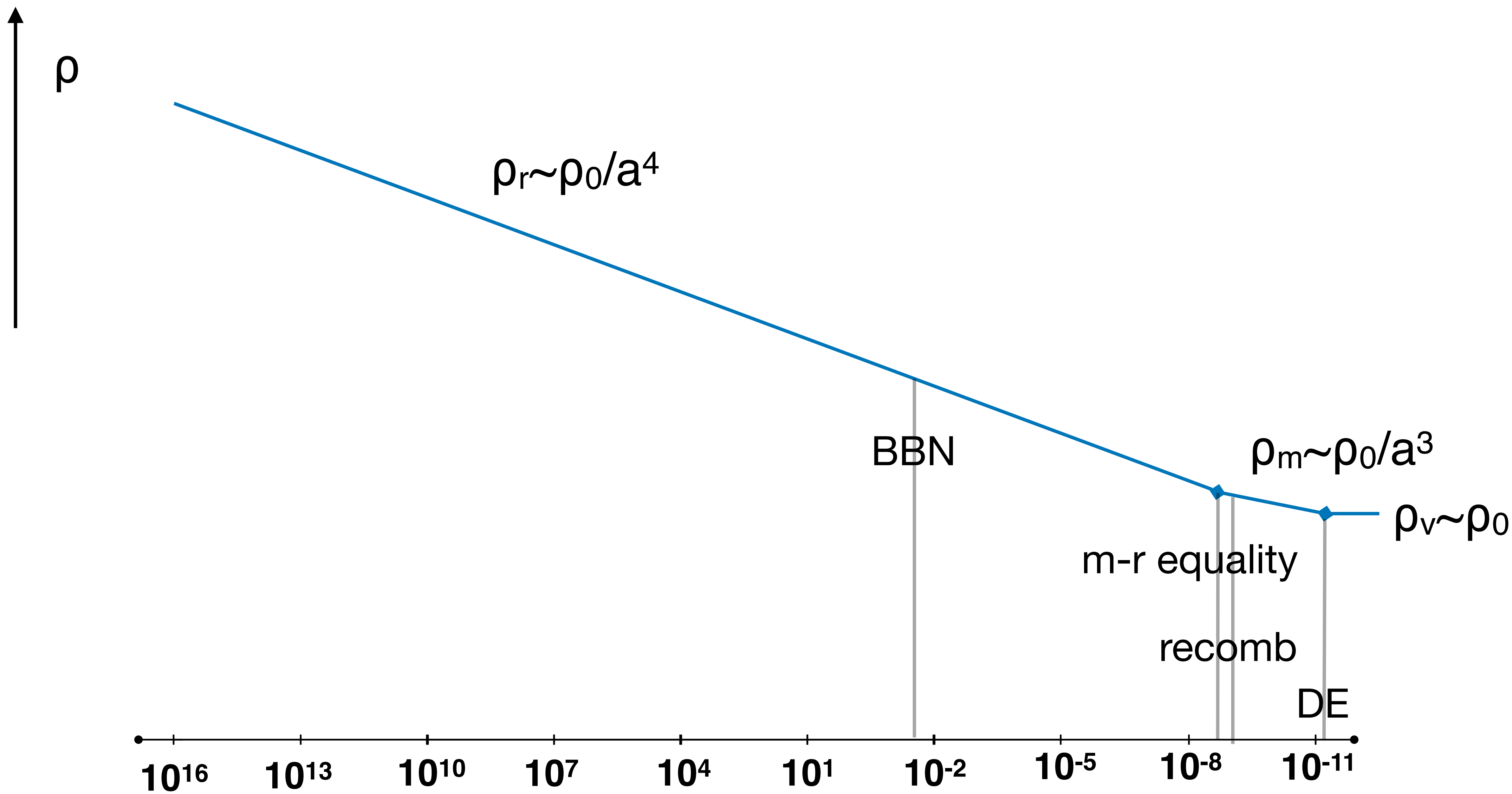

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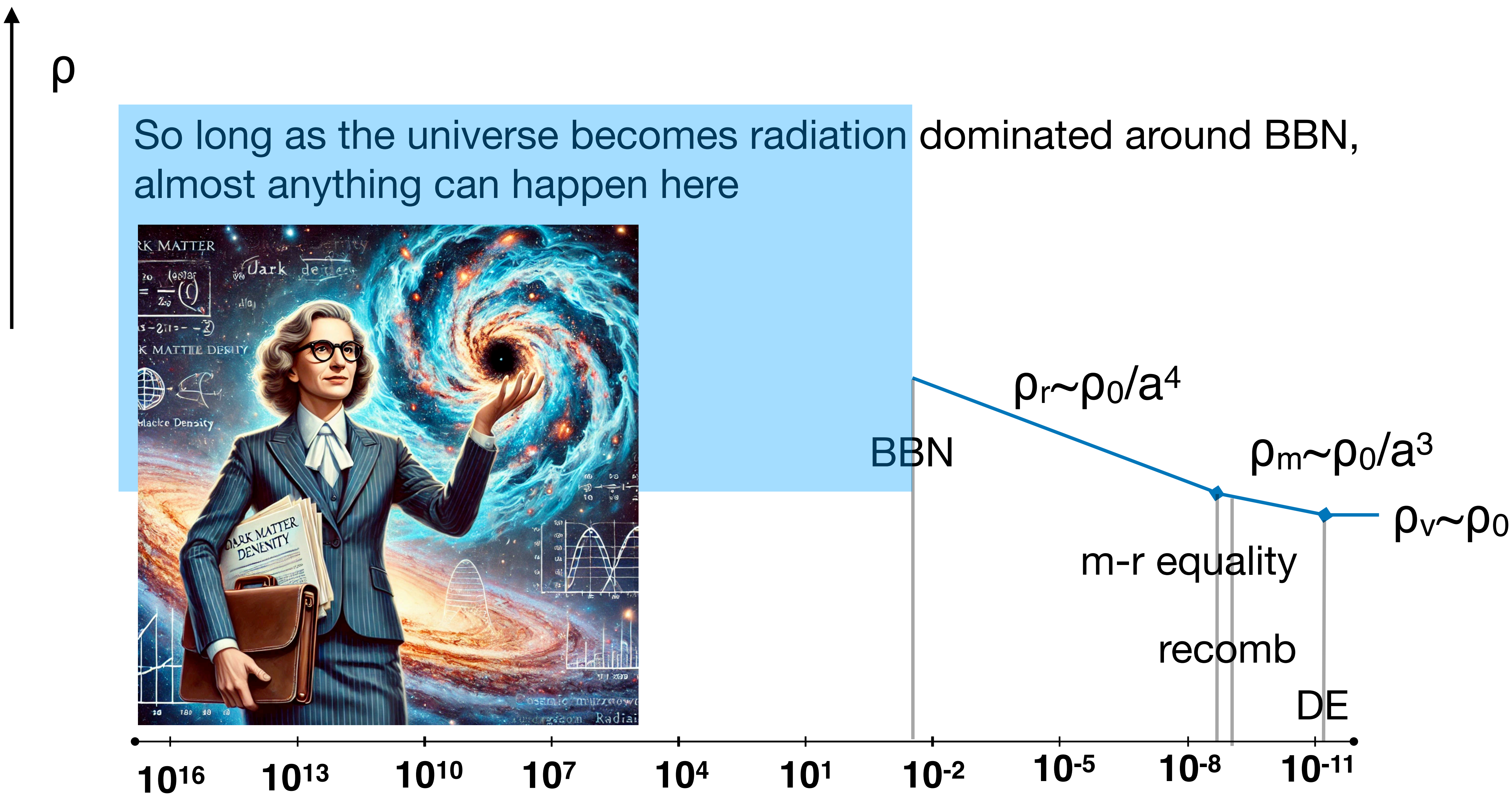
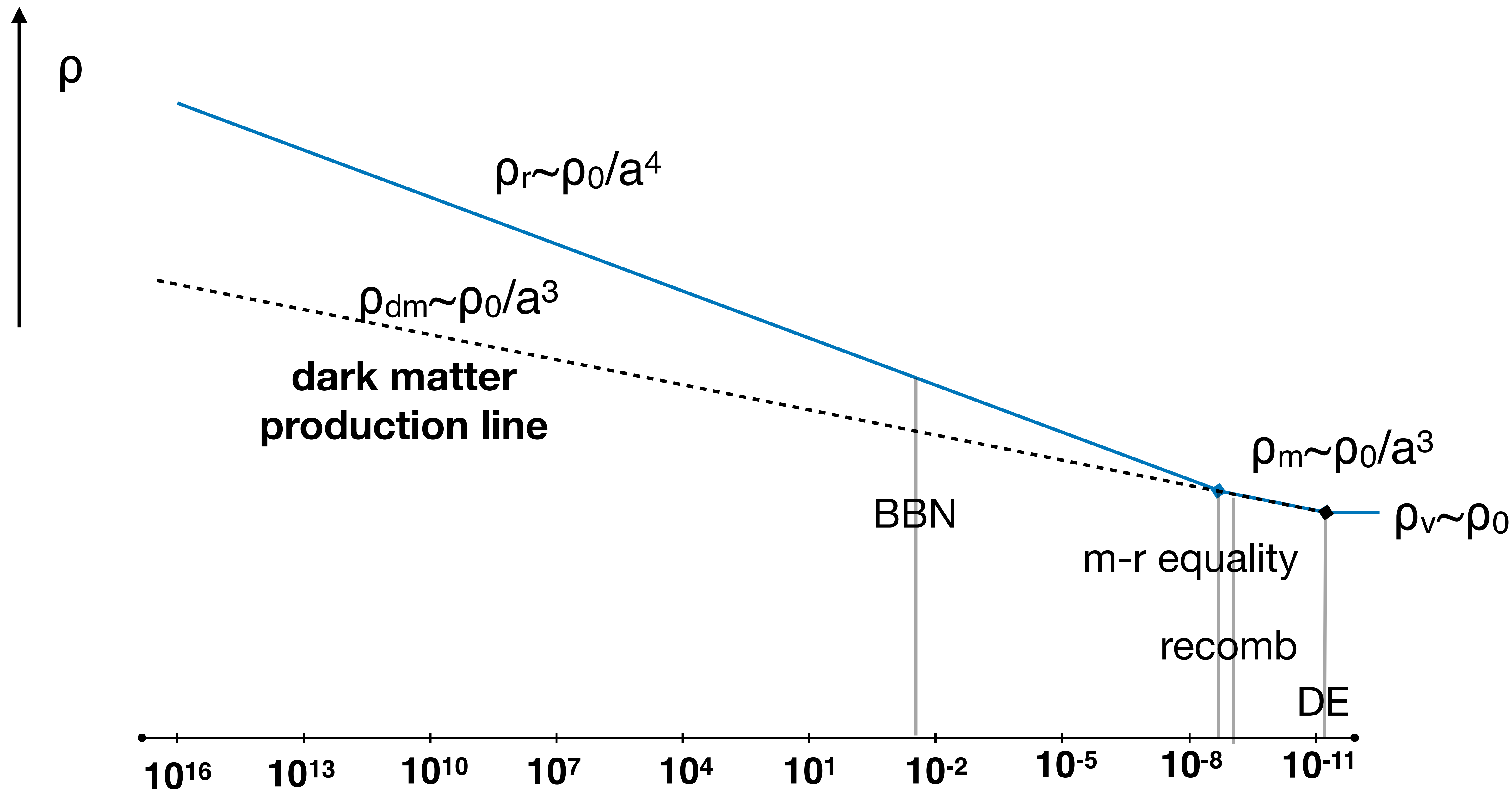
Building a universe



$$T(\text{or } \rho^{1/4}) \propto \frac{1}{a} \text{ in GeV}$$

Building a universe

So long as dark matter appears with a certain lesser abundance, at some point prior to m-r equality, it will come to dominate the mass density of the universe at T~eV



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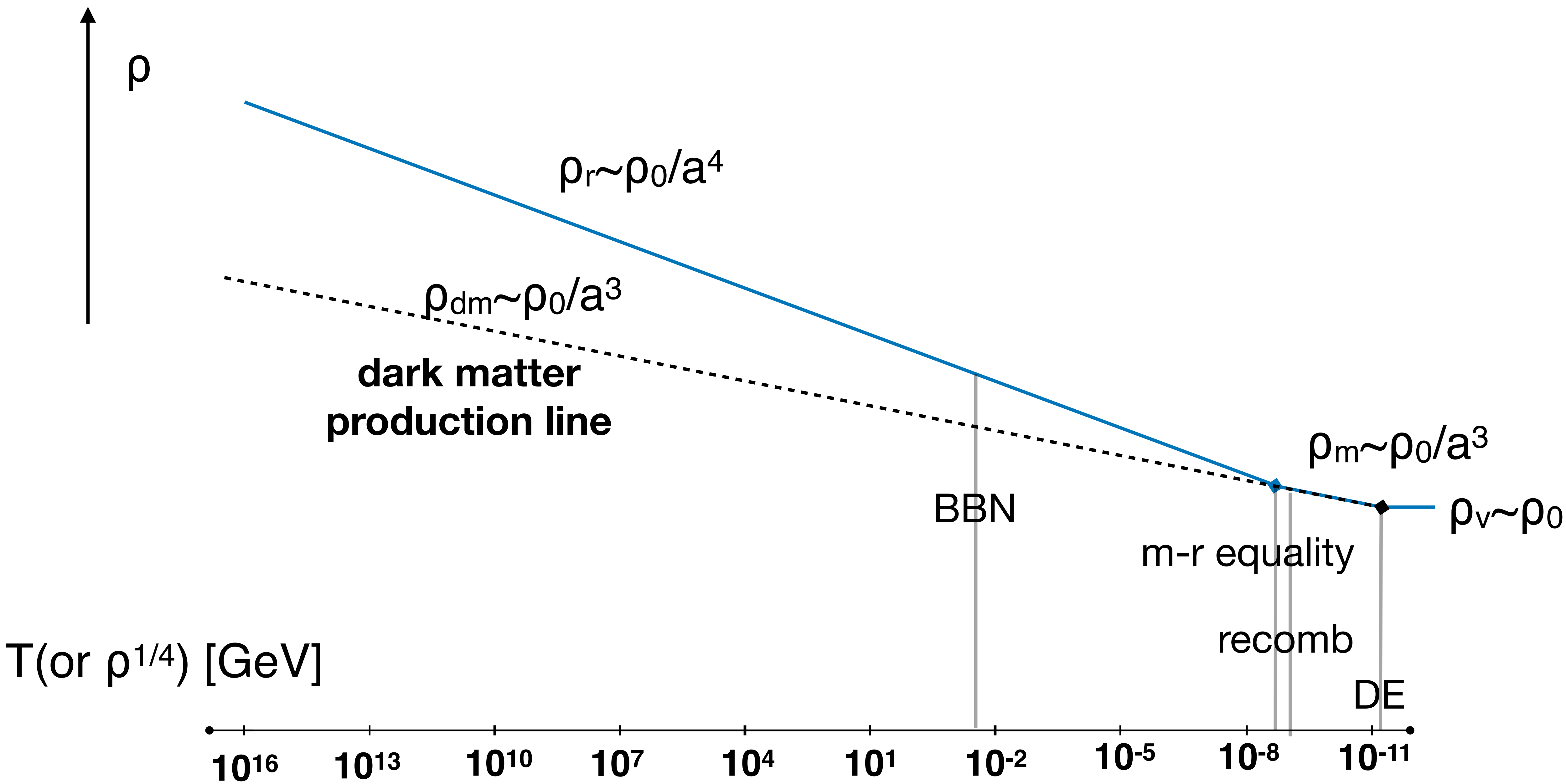


Using $$\rho(T) = \frac{\pi^2 g_*}{30} T^4$$

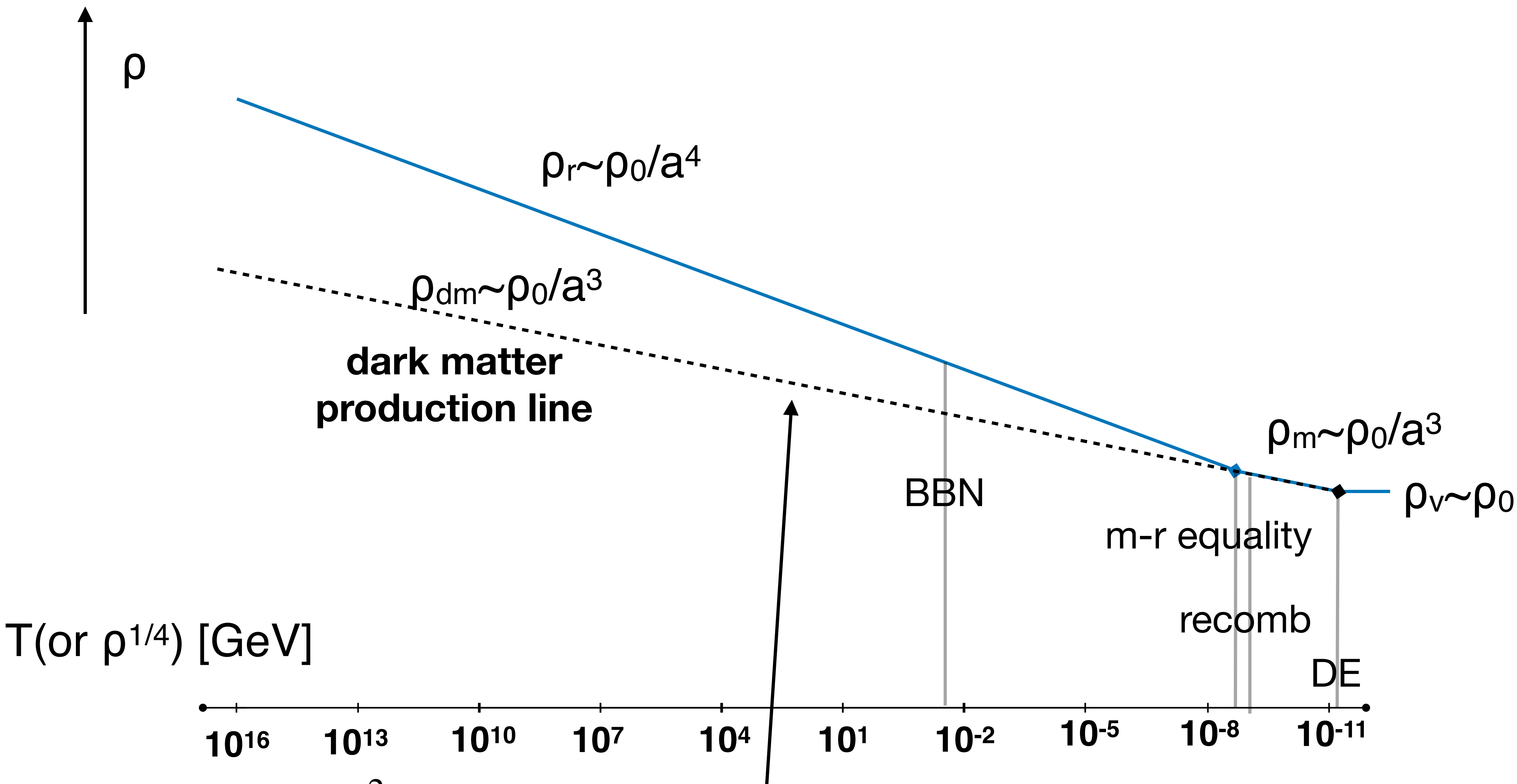
compute the energy density of DM at any temperature $T > T_r$~eV

$$\rho = \rho_0 a^{-3} \quad \text{matter}$$

$a \propto T^{-1}$ in rad. dom. universe

Building a universe

So long as dark matter appears with a certain lesser abundance, at some point prior to m-r equality, it will come to dominate the mass density of the universe at T~eV



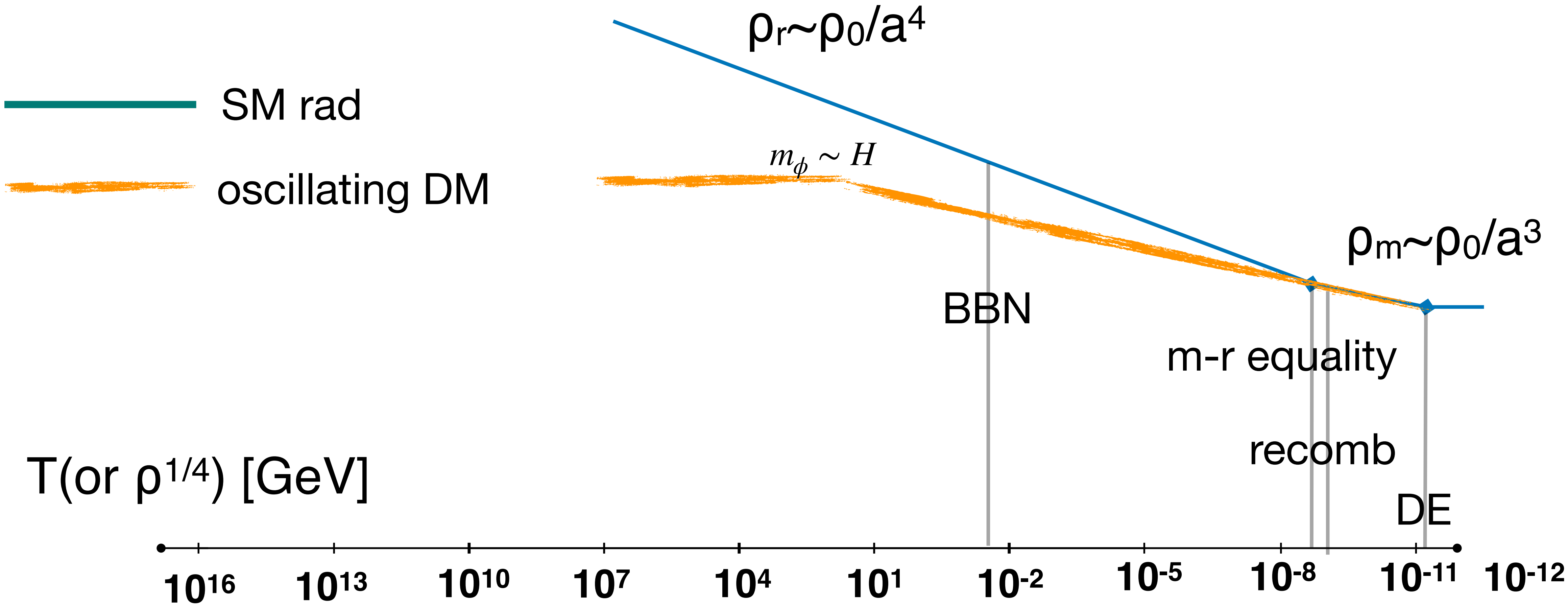
Using $$\rho(T) = \frac{\pi^2 g_*}{30} T^4$$

$$\rho = \rho_0 a^{-3} \quad \text{matter}$$

compute the energy density of DM at any temperature $T > T_r$~eV

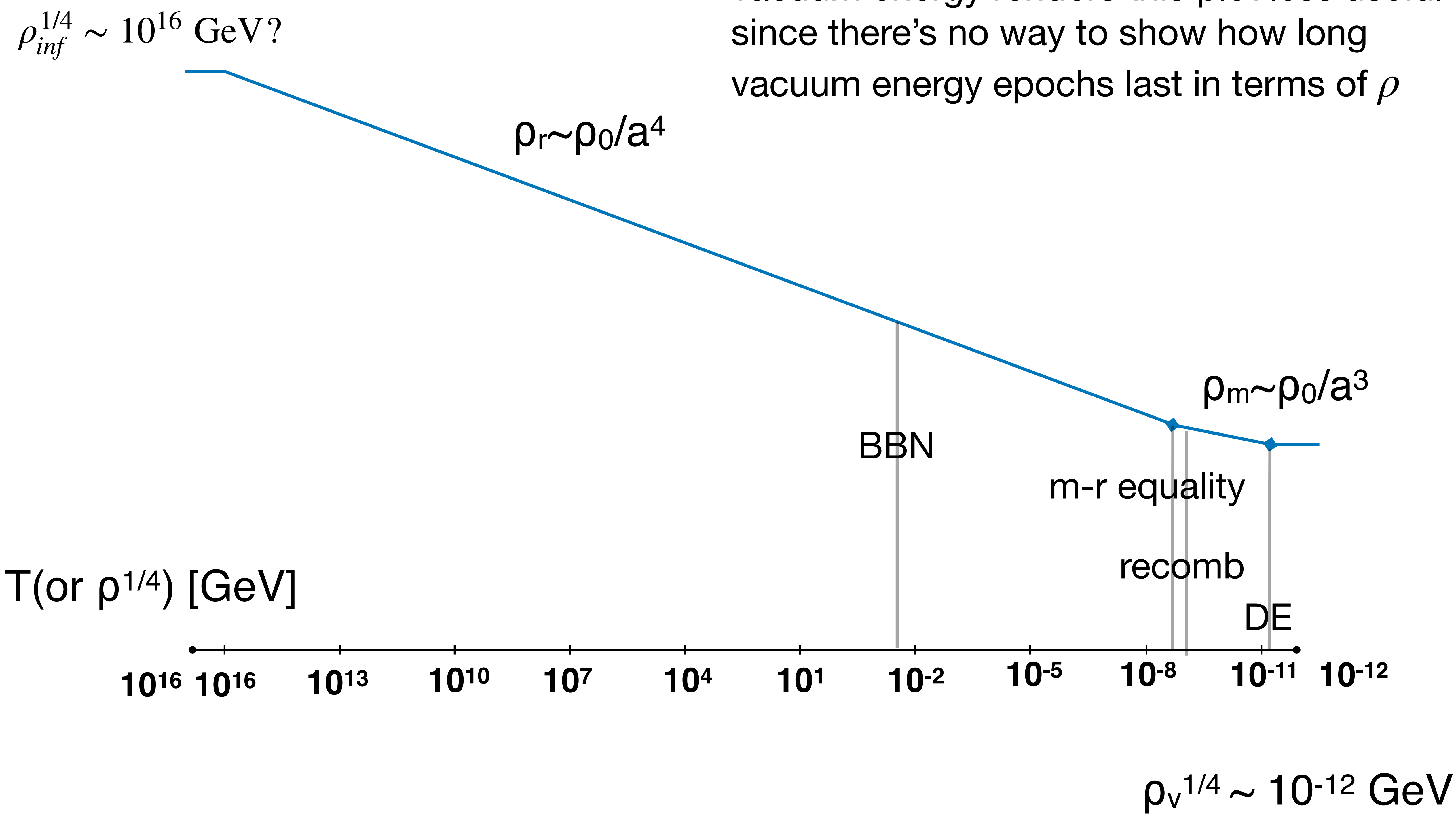
answer: $$\rho_{dm}(T) = \frac{\pi^2 g_*}{30} T^3 T_r$$

Oscillating DM



Quick aside on vacuum energy:

vacuum energy renders this plot less useful since there's no way to show how long vacuum energy epochs last in terms of ρ

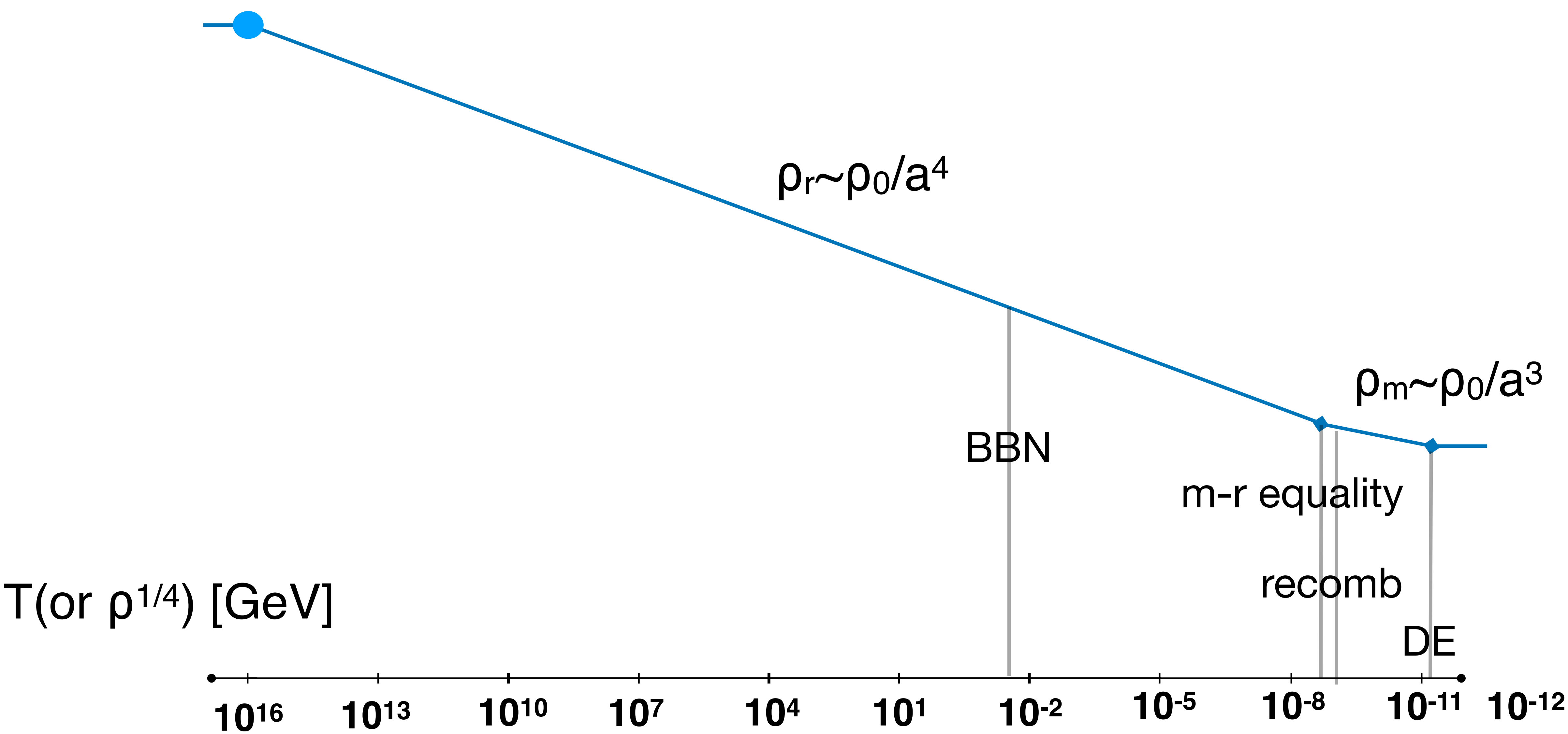


Final note on decaying fields

If we want to consider a decaying field, say the inflaton, we can determine when it decays using its decay width Γ, in most cases, to good approximation we can assume

$$t_{decay} \sim \frac{1}{\Gamma} \sim \frac{1}{H}$$

It can have different branching fractions to (e.g.) a dark sector Γ_{dm}, and the SM Γ_{SM}, but if the decay occurs at $T \gg m_{dm}, m_{sm}$, these will usually form an equilibrium thermal bath.



Capstone Exercise I: Make The Universe

- Introduce all requisite energy densities (as function of T)
- Account for the energy density of the SM, DM
  - Specify decay widths for all fields Γ
- Account for the observed horizon/flatness of our universe
  - (Free to use inflation, initial condition, or something else)
- Bonus*: Account for baryon-anti baryon asymmetry
- Bonus**: What is the smallest possible number of fields/conditions that make our universe?
- Bonus***: How complicated can we make this?

Capstone Exercise I: Make The Universe

$$\begin{cases} \boxed{\rho(T) = \frac{\pi^2}{30} g_* T^4,} \\ s(T) = \frac{4}{3}\frac{\pi^2}{30} g_{*s} T^3 \end{cases}$$

$$g_* = \sum_{i=bosons} g_i \left(\frac{T_i}{T_\gamma}\right)^4 + \frac{7}{8} \sum_{i=\text{ fermions}} g_i \left(\frac{T_i}{T_\gamma}\right)^4$$

$$\boxed{3m_{pl}^2 H^2 = \rho}$$

$$\boxed{\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0}$$

$\rho = \rho_0 a^{-3}$ matter

$\rho = \rho_0 a^{-4}$ radiation

$\rho = \rho_0$ vacuum energy

Quick tip: to relate a decay width to a time during expansion, use the Hubble time
$t_H^{-1} \equiv H = \Gamma_{decay}$

Key temperatures
matter-radiation equality T_r=0.8 eV
baryon asymmetry for BBN T~10^{-1} GeV

max temp from inf? T=10^{16} GeV

Hint for setting dark matter abundance: work backwards from m-r equality T universe where

$$3m_{pl}^2 H^2 = \rho = \frac{\pi^2}{30} g_*^{SM} T^4 \approx \rho_{DM}$$